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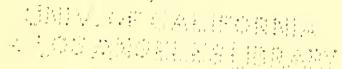
ARITHMETIC

FOR

CARPENTERS AND BUILDERS

BY

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PREFACE

THIS book was written for the purpose of presenting the subject of arithmetic, as used in the daily work of the carpenter and builder, in a simple form. It is intended for the practical man as well as for the beginner and the student. The material is such that the work can be followed successfully by those who have had an eighth-grade education. Upon the completion of this study the student should be prepared to take up the problem of estimating the cost of buildings.

The author has not attempted to treat the subject exhaustively. Though the first chapters may seem elementary to some, they will furnish a much-needed review to others. Geometry is touched upon merely to serve as a foundation for the work in mensuration. Practical applications of geometric truths are emphasized, while rigid proofs and developments are omitted. Two chapters on the steel square, that most useful tool of the carpenter, are included.

Many of the problems do not admit of exact answers for the reason that the judgment of the student plays an important part in the solution. Different results will be obtained, depending upon the lengths and widths of the boards chosen and other similar details. Careful planning for the economical use of material is required, for this is an essential training for the work of the practical carpenter.

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Students will do well to take advantage of this and every similar opportunity to develop their judgment.

It is believed that the reader who follows this text carefully as a guide may learn much that may not strictly be classed as arithmetic. If the manner in which the subject is treated and the many applications made inspire the practical worker and the beginner to seek a better understanding of the fundamentals of carpentry, the author's most sanguine hopes will be justified.

As a text in vocational mathematics it is believed that this little volume should find a place in vocational and industrial schools, trade schools, manual training schools and night schools. It is also well adapted for use in correspondence instruction. The material has been used by the author in the Vocational Courses in Engineering at the Iowa State College and also in the extension classes of the Department of Engineering Extension in various cities in the state of Iowa.

No single book is the entire and original product of one man's mind; it consists rather of accumulated knowledge interpreted and adapted for a particular purpose. The author gratefully acknowledges assistance rendered knowingly and unknowingly by many others. Material borrowed from other sources is acknowledged in the text. The author's appreciative thanks are due his colleague, Mr. H. E. Freund, for reading the manuscript, and to Prof. K. G. Smith for many suggestions and kindly criticisms. Indebtedness is also acknowledged to Sargent & Company for permission to use illustrations of their steel square.

<div align="right">R. Burdette Dale.</div>

Ames, Iowa,
    April, 1915.

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FOR

CARPENTERS AND BUILDERS

CHAPTER I

UNITS OF LENGTH. ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF UNITS OF LENGTH

1. Units of Length. We are all familiar with the common foot rule, by which we measure the length, width or thickness of a board. The *foot* is the unit of length measurement used by English-speaking people, just the same as the dollar is the unit of money in the United States. The foot unit is, however, too long to make small measurements conveniently, therefore it is divided into twelve equal parts called *inches*. With these units of length, the foot and the inch, we can make a great many rough measurements with all the accuracy necessary.

The **foot** is the unit of length measurement.

The **inch** is one-twelfth of a foot.

There are three other units of length based on the foot which are in common use. They are the *yard*, the *rod* and the *mile*.

The **yard** equals three feet or thirty-six inches.

The **rod** equals sixteen and one-half feet or five and one-half yards.

The **mile** equals five thousand two hundred eighty feet.

2. Abbreviations. The word foot is often abbreviated to *ft.*, and the word inch to *in.* The mark (′) also means

FIG. 1.—Section of Timber Framing for Garage.

feet and (″) means inches. These abbreviations may be expressed thus:

$$20 \text{ feet } 4 \text{ inches} = 20 \text{ ft. } 4 \text{ in.} = 20' \ 4''.$$

The abbreviation for yard is *yd.*, and for mile is *mi.*

3. Adding Feet and Inches. You will recollect that you cannot add two unlike quantities and get a result which has any meaning. If you try to add two nails to five screws you have seven things, but they are still two nails and five

screws. If you have two nails and five screws in one box and three nails and four screws in another box and you put them all in the first box, you will have five nails and nine screws in that box. On paper it would look like this:

> 2 nails   5 screws in first box.
> 3 nails   4 screws in second box.
>
> 5 nails   9 screws in one box.

FIG. 2.—Ground Floor Plan of Garage.

When we add feet and inches we must do it in a similar manner.

Example.   7 feet   6 inches
                     10 feet   3 inches

                     17 feet   9 inches.   *Ans.*

The inch is a part of the foot. If we have more than twelve inches, we can change some of the inches to feet and the part left over will be the odd inches. We could not

do this with the nails and screws because a screw is not a part of a nail.   The following example will make this clear.

Example.   Add 9′ 6″ and 10′ 10″.

$$
\begin{array}{rr}
9' & 6'' \\
10' & 10'' \\
\hline
19' & 16'' \\
+\quad 1 & \overline{}12 \\
\hline
20' & 4''
\end{array}
$$

Ans.

Explanation.   Sixteen inches are more than one foot, therefore if we subtract 12″ from 16″ we must also add 1′ to the 19′.   The result will be 20′ 4″ as shown.

4. Subtracting Feet and Inches.   In subtracting feet and inches, we use the same method of combining the units as we did in adding.

Example.   From 12′ 10″ take 4′ 6″.

$$
\begin{array}{rrl}
12' & 10'' & \text{minuend} \\
4' & 6'' & \text{subtrahend} \\
\hline
8' & 4'' & \text{difference.}
\end{array}
$$

Ans.

Notice that in the above example in subtraction the larger of the two numbers is called the **minuend;** the number to be subtracted is called the **subtrahend** and the result is called the **difference.**

Example.   From 9′ 2″ take 6′ 9″.

$$
\begin{array}{rrl}
9' & 2'' = 8' & 14'' \\
& 6' & 9'' \\
\hline
& 2' & 5''
\end{array}
$$

Ans.

Explanation.   Before we can subtract, we must borrow 1′ from 9′ and change it into inches.

5. Multiplying Feet and Inches. Multiplication is a short method of addition. The **multiplicand** is the number to be multiplied and the **multiplier** indicates how many times the multiplicand is to be taken. The result is called the **product**.

To multiply feet and inches, we first multiply the inches and then the feet by the multiplier. If there are 12″ or more in the resulting product, we change the inches to feet so that the result will be expressed in the simplest form.

Example. What is the total length of board required to give 5 pieces each 2′ 6″ long?

$$
\begin{array}{rl}
2' \quad 6'' & \text{multiplicand} \\
5 & \text{multiplier} \\
\hline
10' \quad 30'' & \text{product} \\
+ \quad 2' \quad \overline{24''} & \\
\hline
12' \quad 6'' & \text{simplified product.} \qquad Ans.
\end{array}
$$

Explanation. $5 \times 6'' = 30''$ and $5 \times 2' = 10'$. In order to express this result in its simplest form, we must subtract 24″ from 30″ and add 2′ to 10′. This gives the result 12′ 6″.

6. Division of Feet and Inches. Division is the opposite of multiplication. The number to be divided is called the **dividend.** The number by which the dividend is divided is called the **divisor.** The result is called the **quotient.** To divide feet and inches we first divide the feet and then the inches, expressing the result in its simplest form.

Example. A board 15′ 8″ long is to be cut into four equal lengths. How long will each piece be?

$$
\begin{array}{rl}
4 \text{ divisor })\underline{15' \qquad 8''} & \text{dividend} \\
3\frac{3}{4}' \quad 2'' & \text{quotient} \\
\frac{3}{4}' = 9'' &
\end{array}
$$

Therefore, $3\frac{3}{4}'\ 2'' = 3'\ 11''$ *Ans.*

Explanation. Four goes into 15′ three times and 3 over, or $3\frac{3}{4}'$. One-quarter of a foot is 3″ and three-quarters of a foot is $3\times3''$ or 9″. Four goes into 8″ two times without a remainder. Now add the 9″ which represents the $\frac{3}{4}'$ to 2″ and the simplified quotient is 3′ 11″.

Another very good way to divide feet and inches is first to reduce the feet to inches and then divide.

Example. A board 5′ 10″ long is to be cut in the middle; what is the length of each piece?

$$
\begin{array}{r}
5'\ 10'' = 70'' \\
\times\ 12 \\
\hline
60 \\
+\ 10 \\
\hline
70''
\end{array}
$$

$$
\begin{array}{r}
2\ )\overline{70''} \\
35'' = 2'\ 11''\quad Ans. \\
-\ 24 \\
\hline
11''
\end{array}
$$

Explanation. To reduce 5′ to inches multiply by 12. Then add the 10″ to get 70″. Divide 70″ by 2, the number of equal pieces into which the board is to be cut, to get 35″. This is more than 2′, so we can subtract 24″ from 35″ and we get 11″ as the difference. The simplified form is, therefore, 2′ 11″.

Summary of Chapter I

1. To change **feet to inches,** multiply by 12.

2. To change **inches to feet,** divide by 12.

3. To add measurements, separate the feet and inches and add them separately, changing the result to its simplest form. (Sec. 3.)

4. To subtract feet and inches, subtract each separately and reduce the result to its simplest form. (Sec. 4.)

Fig. 3.

5. To multiply feet and inches, multiply first the inches and then the feet by the multiplier, reducing the result to its simplest form; or reduce the feet to inches and multiply, changing the product again to feet and inches for the answer. (Sec. 5.)

6. To divide feet and inches, divide first the feet and then the inches, reducing the result to its simplest form; or reduce the feet to inches and divide, changing the quotient again to feet and inches for the answer. (Sec. 6.)

PROBLEMS

1. A board is cut up in the factory into three pieces. One piece is 3' 4" long; another is 5' 2" long, and another is 7' 5" long. What was the original length of the board if 2' 1" were wasted?

2. A farmer has a field 36 rods long, and 15 rods wide; how long and how wide is this in feet and inches? What is the total distance around the field in feet and inches?

3. Fig. 1 shows the section of the timber framing for a two story garage. What length of studding must be ordered for the job? To what length must the studding be cut? What is the total distance from the grade line to the ridge of the roof; in other words, what is the distance marked A in the figure?

4. A carpenter wishes to cut four shelves each 2′ 9″ long. He has three boards which he may use. One of these is 10′ long, another 12′ and another 14′. Which board will cut with the least waste? How much waste will there be?
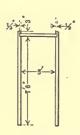
5. Fig. 2 shows the ground floor plan of the garage mentioned in Prob. 3. What is the total length and width of the building?

6. In Fig. 2 how far apart must the windows in the end of the building be placed so that the distances from the edge of each window to the end of the wall and the distance between the windows will be equal?

7. A flag pole is 96′ 3″ long. If it is set 9′ 8″ in the ground, how much of it projects above the ground?

8. A workman in going from his home to the mill walks ten blocks and a half and crosses eleven streets. The blocks are each 300′ long and the streets are 60′ wide. If he makes the trip morning and evening for 300 days in the year, how many miles will he have walked at the end of that time?

FIG. 4.
Door Frame.

9. If it takes the man mentioned in Prob. 8 an average of 17 minutes to make one trip to the mill, how much time will he spend in walking in the course of a year if he makes two trips each day?

10. If telegraph poles are spaced 176′ apart, how many of them are there to the mile?

11. What must be the width of the shop to properly accommodate the buzz saw shown in Fig. 3 if the clearance for the board at the right is the same as that at the left?

12. A shipping box is 77″ long, 43″ wide and 58″ high. Give the dimensions of this box in feet and inches.

13. The steps in a cellar stairs are 2′ 9″ wide and there are 15 of them. What is the total number of lineal feet of stepping required? How many boards 14′ long will be necessary to cut the steps?

14. If 16 sticks each 16′ long and 6 sticks each 10′ long are ordered to cut 16 door frames like the one shown in Fig. 4, how many lineal feet of lumber will be wasted?

FIG. 5.—Cement Walks for Residence and Lawn.

15. The windows in the side of a house 21′ 10″ long are 28″ wide. There are four of them and they are spaced so that the distance between each of them and the distance between the end windows and the ends of the wall are all equal. What is the distance between them?

16. How many lineal feet of two by four edge guides will be required for the cement walks shown in Fig. 5, if all of the work is done in one day so that all the guides are in use at the same time? Allow 50′ for waste and cutting.

FRACTIONS. MIXED NUMBERS. LEAST COMMON DE-NOMINATOR. ADDITION AND SUBTRACTION OF FRACTIONS

7. Fractions. If anything is divided into equal parts we call the parts **fractions.** The fraction may be either one, or several of these parts. Fig. 6 shows a circle made up of segments by the method often used by woodworkers. If you count the segments you will see that there are eight of them. The circle has been divided into eight equal parts and each one of these parts is *one-eighth* of the whole circle. If we select three of these segments they will represent *three-eighths* of the whole.

The expression $8 \div 4 = 2$ means that 8 divided by 4 equals 2. The sign $\div$ means divided by. Now if we replace the dots above and below the line in this sign by the figures, we have $\frac{8}{4} = 2$, which is a simpler way of expressing the same thing. But this $\frac{8}{4}$ is a fraction. Thus we see that a fraction is another way of expressing division, and it means that the number above the line is to be divided by the number below the line.

8. Definitions. A **fraction** is one or more of the equal parts into which anything may be divided. For example three-eighths means that the whole is divided into eight parts and three of these are taken. A fraction also indicates division.

A fraction has two parts called **terms.** One is the

denominator which is written below the line and this tells into how many parts the whole has been divided. The **numerator** or the number above the line tells how many of these parts are taken.

9. Reading and Writing Fractions. To pronounce a fraction, name the numerator first and then the denominator followed by the ending *th* or *ths*. When the numbers **2, 3**

FIG. 6.—Circle made up of Segments.

and 4 are used for the denominator we say *one-half*, *one-third* or *one-quarter*. Also when the number of the denominator ends in 1, 2 or 3, we say one twenty-first, one thirty-second or five fifty-thirds, as the case may be.

To write a fraction, place the numerator over the denominator with a line between. For example, three-eighths is written $\frac{3}{8}$; seven-sixteenths is written $\frac{7}{16}$.

10. Proper and Improper Fractions. A **proper fraction** is one the numerator of which *is less than* the denominator. The value of a proper fraction is less than unity or one.

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{8}$ and $\frac{7}{16}$ are all proper fractions because the numerators are less than the denominators.

An **improper fraction** is one the numerator of which *is equal to or greater than* the denominator. If the numerator is equal to the denominator, the value of the fraction is unity or one, for the reason that all of the parts into which the whole has been divided are taken. For example, $\frac{2}{2}$, $\frac{4}{4}$, $\frac{8}{8}$ and $\frac{16}{16}$ are improper fractions the value of each of which is 1. If the numerator of the fraction is greater than the denominator, the value of the fraction is greater than one.

FIG. 7.—Portion of an Ordinary Rule.

For example, $\frac{3}{2}$, $\frac{9}{4}$ and $\frac{11}{8}$ are all improper fractions, the value of each of which is greater than 1.

11. Mixed Numbers. A **mixed number** is a number composed of a whole number and a fraction written together. For example, $5\frac{1}{2}$ is a mixed number and denotes five and a half units. If the units are feet, this would mean five and one-half feet.

12. Fractions of an Inch. If you will examine the ordinary rule carefully, a representation of which is shown in Fig. 7, you will notice that the inch is divided first into two parts or fractions, each being one-half of an inch. The halves are again equally divided into quarters of an inch and these again into eighths and sixteenths of an inch. For very accurate measurements some rules are divided into thirty-seconds and sixty-fourths of an inch.

13. Reduction of Fractions to Lower Terms. Everyone knows that $\frac{1}{2}'' + \frac{1}{2}'' = \frac{2}{2}'' = 1''$. We can also say,

$$\frac{1}{4}'' + \frac{1}{4}'' = \frac{2}{4}'' = \frac{1}{2}'' \text{ and } \frac{1}{8}'' + \frac{1}{8}'' = \frac{2}{8}'' = \frac{1}{4}''.$$

This shows that the same value can be expressed by fractions in different ways. In spite of this fact we do not usually say two-eighths of an inch when we mean one-quarter of an inch, although they both have the same value. We are obliged to say three-sixteenths of an inch, but we usually say one-eighth rather than two-sixteenths. Unless it is necessary to do otherwise we like to reduce the fraction to its lowest terms.

Example. Reduce $\frac{12}{32}''$ to its lowest terms.

$$\frac{12}{32}'' \div \frac{4}{4} = \frac{3}{8}'' \qquad\qquad Ans.$$

Explanation. Divide both the numerator and the denominator of the fraction by the same number and use the largest number that will go into both an even number of times. In this case 4 is the largest number which will go into both 12 and 32.

Thus we see that if we divide both the numerator and the denominator of the fraction by the same number, we change the form of the fraction but we do not change its value.

14. Reductions of Fractions to Higher Terms. If we multiply both the numerator and denominator of a fraction by the same number, we will change its form but we will not change its value. We can, therefore, reduce a fraction to higher terms. In this way we can increase the denominator of any fraction.

Example. Change $\frac{3}{8}''$ to sixteenths of an inch.

$$\frac{3}{8}'' \times \frac{2}{2} = \frac{6}{16}'' \qquad\qquad Ans.$$

Explanation. To change eighths to sixteenths we must multiply the denominator by 2, but we cannot do this without changing the value of the fraction unless we also multiply the numerator by 2. Multiplying both the numerator and denominator by 2 gives the required result, $\frac{6}{16}''$.

Example. Change $\frac{5}{8}''$ to thirty-seconds.

$$\frac{5}{8}'' \times \frac{4}{4} = \frac{20}{32}'' \qquad\qquad Ans.$$

Explanation. We must multiply the denominator 8 by 4 to get 32. If we multiply the denominator by 4 we must also multiply the numerator by 4. The result is $\frac{20}{32}''$.

15. Changing Whole Numbers to Improper Fractions. A whole number can be changed into the form of a fraction by using the number 1 for a denominator. This does not change the value of the number. For example, the whole number 6 is equal to $\frac{6}{1}$. We can multiply both the numerator and denominator of this fraction by the same number without changing its value.

Example. Change the whole number 6 to eighths.

$$\frac{6}{1} \times \frac{8}{8} = \frac{48}{8} \qquad\qquad Ans.$$

Explanation. We first changed the whole number into the form of a fraction by using the denominator 1. To reduce $\frac{6}{1}$ to 8ths we must multiply both numerator and denominator by 8. The result is an improper fraction, $\frac{48}{8}$.

16. Changing Mixed Numbers. A mixed number can be changed to an improper fraction by multiplying the whole number by the denominator of the fraction, adding to this the numerator of the fraction and placing the sum over the denominator. The result will be an improper fraction which is equivalent to the mixed number.

Example. Change $5\frac{1}{2}$ to an improper fraction having 2 for a denominator.

$$5\frac{1}{2} = \frac{10}{2} + \frac{1}{2} = \frac{11}{2} \qquad\qquad Ans.$$

Explanation. In one unit there are 2 halves; therefore, in 5 units there are 5 times 2 halves or 10 halves. Adding the $\frac{1}{2}$ to the $\frac{10}{2}$ we get $\frac{11}{2}$ for the result. If you will examine the rule given above you will see that it has been followed in the solution of this example.

17. Changing Improper Fractions. Changing an improper fraction to a mixed number is just the reverse of changing a mixed number to an improper fraction. The process is as follows: Divide the numerator by the denominator to find the whole number and annex to this the fraction obtained by using the remainder for a numerator and the denominator of the improper fraction for a denominator. The result is a mixed number.

Example. Change $\frac{39}{8}$ to a mixed number.

$$8\,)39(\,4\tfrac{7}{8} \qquad\qquad Ans.$$
$$\underline{32}$$
$$7$$

Explanation. Dividing the numerator by the denominator, 8 goes into 39 four times with a remainder of 7. Placing this remainder over the denominator and annexing this fraction to the whole number in the quotient, we have $4\frac{7}{8}$ for the result.

18. Least Common Denominator. Suppose you are required to find the combined thickness of two boards one of which is $\frac{1}{4}$ of an inch thick and the other is $\frac{5}{8}$ of an inch thick. If you measured these two boards together with your rule you would see at once that the combined width is $\frac{7}{8}$ of an inch. How would you add these two fractions
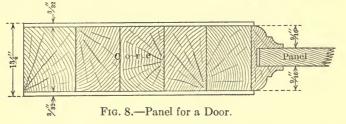
on paper?   You will see that since the answer is $\frac{7}{8}$, you must change both of the fractions to eighths before you can add them, because you cannot add two things that are not of the same kind.   You must, therefore, reduce the fractions to a common denominator.

Example.   What is the sum of $\frac{1}{4}''$ and $\frac{5}{8}''$?

$$\frac{1}{4}'' \times \frac{2}{2} = \frac{2}{8}''$$
$$\frac{5}{8}'' \qquad = \frac{5}{8}''$$
$$\overline{\qquad\quad \frac{7}{8}''} \qquad\qquad Ans.$$

Explanation.   Change $\frac{1}{4}''$ to eighths by multiplying both numerator and denominator by 2.   This gives $\frac{2}{8}''$.   The

FIG. 8.—Panel for a Door.

other fraction is already expressed in eighths so that we can now add the numerators and place their sum over the common denominator.

The **Least Common Denominator** is the smallest denominator to which all of several fractions may be changed. The abbreviation for the least common denominator is L.C.D.

If you are working with ordinary fractions of an inch, it is not difficult to find the L.C.D. provided all of the fractions are first reduced to their lowest terms.   The ordinary fractions of an inch commonly used are halves, quarters, eighths, sixteenths, thirty-seconds and sixty-fourths.   The L.C.D. of a number of fractions having these denomina-

tors will be the largest denominator which occurs in the group.

Example. Find the L.C.D. of the following fractions: $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, $\frac{9}{32}$, $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{5}{16}$.

$$\frac{5}{8} \times \frac{4}{4} = \frac{20}{32}; \quad \frac{7}{16} \times \frac{2}{2} = \frac{14}{32}; \quad \frac{3}{4} \times \frac{8}{8} = \frac{24}{32}; \quad \frac{9}{32};$$
$$\frac{1}{4} \times \frac{8}{8} = \frac{8}{32}; \quad \frac{1}{2} \times \frac{16}{16} = \frac{16}{32}; \quad \frac{5}{16} \times \frac{2}{2} = \frac{10}{32}. \qquad Ans.$$

Explanation. Since all of the fractions are expressed in their lowest terms, we can determine the L.C.D. by in-

Fig. 9.—Board Grooved for Splines.

spection. It will be the largest number among the denominators which, in this case, is 32. Now taking each fraction in turn we divide its denominator mentally into 32 to get the number 'by which it must be multiplied. We then multiply both the numerator and denominator by this number.

19. To Find the Least Common Denominator. Now and then it is necessary to find the L.C.D. of several fractions when it cannot be determined by inspection. In that case, we first place the denominators in a row, separating them by dashes. Then select the smallest number, other than 1, which will be exactly contained into two or more

of the denominators. Draw a line under this row and perform the division, writing the quotients below the line. Also write below this line any numbers which are not divisible by the divisor. Then repeat the operation, selecting a new divisor that will be exactly contained into two or more of these new numbers. Continue this process until no number will exactly divide more than one of the numbers below the line. The product of all of the divisors and all of the numbers left in the last line of quotients is the L.C.D.

Example. Find the L.C.D. of $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{3}{16}$.

$$
\begin{array}{r}
2\,)\overline{4\text{—}3\text{—}9\text{—}16} \\
2\,)\overline{2\text{—}3\text{—}9\text{—}\ 8} \\
3\,)\overline{1\text{—}3\text{—}9\text{—}\ 4} \\
\overline{1\text{—}1\text{—}3\text{—}\ 4}
\end{array}
$$

L.C.D. $= 2\times2\times3\times3\times4 = 144$ *Ans.*

Explanation. First place the denominators of the fractions in a row, separating them by dashes. Then select the number 2 for a divisor which will be contained into both 4 and 16. Perform the division and write down the quotients, bringing down the 3 and 9 also. Select 2 again and divide it into 2 and 8, writing down the quotients and the 3 and the 9 as before. The number 2 will no longer be contained into more than one of the numbers remaining; hence, we will try 3. This will go into 3 and 9, and after writing the quotients of this division we find that no number will be contained into more than one of the numbers remaining. To find the L.C.D. then we multiply all of the divisors and all of the remaining numbers together. This gives the result 144, which is the smallest number into which all of the denominators will be exactly contained without a remainder.

20. To Reduce to the Least Common Denominator.
After we have found the L.C.D. we next reduce each fraction
in turn to that denominator. To do this we multiply both
the numerator and the denominator by such a number as
will raise the fraction to the required L.C.D. To find the
number by which to multiply, divide the denominator of
the fraction into the L.C.D. The quotient will be the
required number.

Fig. 10.—Section of Partition Material.

Example. Reduce the fractions in the previous example
to their L.C.D.

$$144 \div 4 = 36 \qquad \tfrac{1}{4} \times \tfrac{36}{36} = \tfrac{36}{144}$$
$$144 \div 3 = 48 \qquad \tfrac{2}{3} \times \tfrac{48}{48} = \tfrac{96}{144}$$
$$144 \div 9 = 16 \qquad \tfrac{5}{9} \times \tfrac{16}{16} = \tfrac{80}{144}$$
$$144 \div 16 = 9 \qquad \tfrac{3}{16} \times \tfrac{9}{9} = \tfrac{27}{144} \qquad Ans.$$

Explanation. In the previous example we found that
the L.C.D. of $\tfrac{1}{4}$, $\tfrac{2}{3}$, $\tfrac{5}{9}$, and $\tfrac{3}{16}$ was 144. We must then divide
each denominator into this number in turn and multiply
both numerator and denominator of the fraction by this
quotient. Taking the first fraction, 4 will go into 144
thirty-six times. Multiplying both the numerator and the
denominator of the fraction $\tfrac{1}{4}$ by 36 we get $\tfrac{36}{144}$ for the
result. We proceed in exactly the same way for each of the
other fractions.

21. Adding Fractions. To add fractions change them to their L.C.D., then add the numerators and place their sum over the L.C.D. Reduce the resulting fraction to its lowest terms if necessary.

Example. Add the following fractions of an inch: $\frac{3}{4}''$, $\frac{5}{16}''$, $\frac{1}{2}''$, $\frac{3}{8}''$ and $\frac{9}{32}''$.

$$\frac{3''}{4} \times \frac{8}{8} = \frac{24}{32''}$$

$$\frac{5''}{16} \times \frac{2}{2} = \frac{10}{32''}$$

$$\frac{1''}{2} \times \frac{16}{16} = \frac{16}{32''}$$

$$\frac{3''}{8} \times \frac{4}{4} = \frac{12}{32''}$$

$$\frac{9''}{32} \times \frac{1}{1} = \frac{9}{32''}$$

$$\frac{71}{}$$

$$32\,)71\,(\,2\tfrac{7}{32} \qquad Ans.$$

$$\frac{64}{7}$$

Explanation. By inspection we see that 32 is the L.C.D. Changing each fraction to this L.C.D. and adding the numerators, we have $\frac{71}{32}''$. Reducing this improper fraction to a mixed number gives $2\frac{7}{32}''$ as the result.

22. Subtracting Fractions. To subtract fractions we must first change them to their L.C.D. the same as we did in adding fractions. Then subtract the numerator of the

subtrahend from the numerator of the minuend and write
the result over the common denominator. If necessary,
reduce the result to its lowest terms.

Example. Subtract $\frac{3}{32}''$ from $\frac{7}{8}''$.

$$\frac{7}{8} = \frac{28}{32}$$
$$\frac{3}{32}$$
$$\frac{25''}{32}$$ *Ans.*

Explanation. Changing $\frac{7}{8}''$ to thirty-seconds, we multiply
both numerator and denominator by 4. This gives $\frac{28}{32}''$.
We can now subtract $\frac{3}{32}''$ and obtain $\frac{25}{32}''$ for the result.

SUMMARY OF CHAPTER II

7. The numerator and the denominator of a fraction *may both be divided by the same number* without changing the value of the fraction.   (Sec. 13.)

8. The numerator and the denominator of a fraction *may both be multiplied by the same number* without changing the value of the fraction.   (Sec. 14.)

FIG. 11.—Mortise and Tenon Joint for Table Leg.

9. **To reduce a fraction to its lowest terms,** divide both the numerator and the denominator by the largest number which will exactly go into each of them.   (Sec. 13.)

10. **To reduce a fraction to higher terms,** multiply both the numerator and denominator by the number which will give the required denominator.   (Sec. 14.)

11. **To change a whole number to an improper fraction,** having a given denominator, supply the denominator 1 to

the whole number and multiply both numerator and denominator of this resulting fraction by the number which will give the required denominator.   (Sec. 15.)

12. **To change a mixed number to an improper fraction,** multiply the whole number by the denominator of the fraction, add to this the numerator of the fraction and place the sum over the denominator.   (Sec. 16.)

13. **To change an improper fraction to a mixed number,** divide the numerator by the denominator to find the whole

Fig. 12.—Dovetail Drawer Joint.

number and annex to this the fraction obtained by using the remainder for a numerator and the denominator of the improper fraction for a denominator.   (Sec. 17.)

14. **To find the least common denominator** of several fractions, find the least number into which each of the denominators will be exactly contained without a remainder. (Sec. 18 and 19.)

15. **To reduce several fractions to their least common denominator,** multiply both the numerator and the denominator of each fraction by such a number as will give the required L.C.D.   (Sec. 20.)

16. **To add fractions,** first reduce them to their L.C.D.,

then add the numerators and place their sum over the common denominator. Simplify this result, if possible. (Sec. 21.)

17. **To subtract fractions,** first reduce them to their L.C.D., then subtract their numerators and place the result over the common denominator. Simplify, if possible. (Sec. 22.)

Fig. 13.—Dovetail Mortise Joint.

PROBLEMS

17. (a) Change $\frac{39}{16}''$ to a mixed number.

    (b) Change $9\frac{7}{8}''$ to an improper fraction.

18. Which is greater, $\frac{13}{16}''$ or $\frac{7}{8}''$?

19. Reduce the following fractions to their lowest terms: $\frac{18}{32}$, $\frac{4}{16}$, $\frac{12}{32}$, $\frac{8}{12}$.

20. Change the following fractions to their L.C.D.: $\frac{3}{32}''$, $\frac{29}{64}''$, $\frac{7}{8}''$, $\frac{3}{16}''$.

21. Add the following fractions: $\frac{3}{16}''$, $\frac{7}{8}''$, $\frac{3}{4}''$ and $\frac{9}{16}''$.

22. From $\frac{7}{8}''$ take $\frac{9}{16}''$.

23. Fig. 8 shows a panel for a door. What is the thickness of the panel?

24. The door stile shown in Fig. 8 is to be $1\frac{3}{4}''$ thick. If the veneers are each $\frac{3}{32}''$ thick, how thick must the core be?

25. A drawing board is built of three plies; one is $\frac{3}{8}''$ thick, one is $\frac{11}{16}''$ thick and one is $\frac{3}{32}''$ thick. What is the total thickness of the board?

26. A dressed board is required $\frac{25}{32}''$ thick. How much must be planed off from a rough board $1''$ thick to give the required thickness?

27. A floor is laid by first putting down the rough sheathing which is $\frac{7}{8}''$ thick. On this are placed furring strips $\frac{7}{8}''$ thick and then the top or finish floor is laid which consists of boards $\frac{13}{16}''$ thick. What is the total thickness of this floor?

28. Fig. 9 shows a board grooved for splines. If a $\frac{7}{16}'' \times \frac{7}{8}''$ spline is to be used, what will be the width of the projection A on each side of the groove?

29. Fig. 10 shows 1×4 partition material in section. Give the missing dimensions indicated by the letters A, B, C and D.

30. Flooring $1\frac{1}{2}''$ nominal size is worked to $1\frac{11}{32}''$ in the mill. How much is it scant?

31. Fig. 11 shows a mortise and tenon joint for a table leg. How far back from the face of the leg must the outside edge of the mortise be? In other words, what is the distance A?

32. What is the missing dimension on the dovetail drawer joint indicated by the letter A in Fig. 12?

33. What must be the size of the square key used for the dovetail mortise joint shown in Fig. 13?

34. A lumber dealer has in stock 2000 bundles of shingles. Of these 400 bundles are graded Extra Clear and 1600 are Extra *A*. Each grade is what fractional part of the whole stock? Give the fractions in their lowest terms.

CHAPTER III

MIXED NUMBERS. ADDITION, SUBTRACTION AND MULTIPLICATION OF MIXED NUMBERS. CANCELLATION

23. Working with Mixed Numbers. In performing operations with mixed numbers, there are two possible methods of procedure; either the whole numbers and the fractions may each be handled separately or the mixed numbers may be reduced to improper fractions and the operations performed as though working directly with fractions. In general, the first method is used when adding or subtracting mixed numbers and the second method is used when multiplying or dividing them.

24. Adding Mixed Numbers. It is often necessary to add mixed numbers. Although this operation is somewhat long it is not at all difficult. To add mixed numbers we first add the whole numbers and then the fractions, reducing the final result to its lowest terms.

Example. What is the sum of the following dimensions: $6\frac{7}{8}''$, $3\frac{3}{16}''$, $9\frac{11}{16}''$, $6\frac{1}{2}''$ and $4\frac{3}{32}''$?

$$6\frac{7}{8}''\text{ (Reduce fractions to L.C.D.)} = \frac{28}{32}$$

$$3\frac{3}{16}'' \qquad\qquad\qquad = \frac{6}{32}$$

$$9\frac{11}{16}'' \qquad\qquad\qquad = \frac{22}{32}$$

$$6\tfrac{1}{2}'' \qquad\qquad = \frac{16}{32}$$

$$4\tfrac{3}{32}'' \qquad\qquad = \frac{3}{32}$$

$$\overline{28} \qquad\qquad\qquad \overline{75}$$
$$+ \qquad\qquad\qquad\qquad \overline{32}$$
$$2\tfrac{11}{32}$$

$$\overline{30\tfrac{11}{32}''} \;\; Ans. \qquad \text{Reducing} = 32\;)75(\;\; 2\tfrac{11}{32}''$$
$$\qquad\qquad\qquad\qquad\qquad\qquad \frac{64}{11}$$

Explanation. The first step is to write the numbers in a vertical column for adding. It is then necessary to reduce the fractions to a common denominator before we can add them. The common denominator in this case is easily seen to be 32. We must, then, multiply both the numerator and denominator of each fraction by such a number as will make the denominator of the new fraction 32. After this has been done we add the numerators and place their sum over the common denominator, finally reducing the resulting improper fraction to a mixed number, which is $2\tfrac{11}{32}''$. We next add the whole numbers together to get 28 and to this we add the number obtained by adding the fractions. This gives $30\tfrac{11}{32}''$ as the final result.

25. Subtracting Mixed Numbers. To subtract mixed numbers we treat the whole numbers and the fractions separately, just as we did in adding them. We subtract the fractions and the whole numbers separately and reduce the resulting expression to its lowest terms.

Example. How much more is $9\tfrac{13}{16}''$ than $7\tfrac{5}{8}''$?

$$9\tfrac{13}{16}'' \;\; \text{(Reduce fractions to L.C.D.)} \; = 9\tfrac{13}{16}''$$
$$7\tfrac{5}{8}'' \qquad\qquad\qquad\qquad\qquad\qquad = 7\tfrac{10}{16}''$$
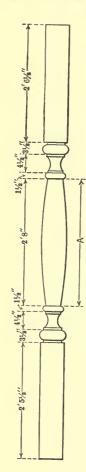$$\overline{2\tfrac{3}{16}''} \;\; Ans.$$

FIG. 14.—Turned Porch Post.

Explanation. The first step, just as in adding fractions, is to write the figures in a vertical column. We must then reduce the fractions to a common denominator before we can subtract them. The L.C.D. in this case is 16. We do not have to do anything with the $\frac{13}{16}''$ but we do have to change $\frac{5}{8}''$ to sixteenths. Multiplying both the numerator and the denominator of this fraction by 2 gives $\frac{10}{16}$. We can now subtract both the fractions and the whole numbers to get the result, $2\frac{13}{16}''$, which tells how many inches longer is a stick which measures $9\frac{13}{16}''$ than a stick which measures $7\frac{5}{8}''$.

Sometimes in subtracting mixed numbers we find that the fraction in the subtrahend (the number to be taken away) is larger than the fraction in the minuend (the number from which we subtract). In this case we may borrow one whole unit or 1 from the whole number of the minuend and add it to the fraction of the minuend after it has been reduced to the same denominator. This makes an improper fraction out of it and we can then subtract the other fraction from it.

Example. How much wider is a board which measures $8\frac{1}{4}''$ than a board which measures $5\frac{7}{8}''$?

$$8\frac{1}{4}'' = 8\frac{2}{8}'' = 7\frac{10}{8}''$$
$$5\frac{7}{8}'' \qquad\ = 5\frac{7}{8}''$$
$$\overline{\qquad\qquad 2\frac{3}{8}''} \qquad\qquad Ans.$$

Explanation. Here we cannot take $\frac{7}{8}''$ from $\frac{1}{4}''$ until we change $\frac{1}{4}''$ to eighths and add the 1 or $\frac{8}{8}$ to it which we have borrowed from the whole number 8. This makes the minuend read $7\frac{10}{8}''$ and we can now proceed to subtract. The result is $2\frac{3}{8}''$, which tells how much wider the one board is than the other.

If the minuend happens to be a whole number without a fraction we can borrow 1 and change it into a fraction as we did in the previous example.

Example. What is the difference between $5\frac{7}{8}''$ and $8''$?

$$8'' = 7\frac{8}{8}''$$
$$5\frac{7}{8}''$$
$$\overline{2\frac{1}{8}''} \qquad \qquad Ans.$$

26. Multiplication of Fractions and Mixed Numbers. We have already learned that multiplication is a short method of addition. In multiplying fractions and whole numbers, fractions and fractions, fractions and mixed numbers, and mixed numbers together, certain precautions must be taken which lead to speed and accuracy.

27. The Product of a Whole Number and a Fraction. To multiply a whole number by a fraction we multiply the number by the numerator and divide this product by the denominator, or if it is possible without a remainder, simply divide the denominator by the number.

Multiplying a whole number by a fraction is the same thing as multiplying the fraction by the whole number: thus, $5 \times \frac{5}{7}$ is the same thing as $\frac{5}{7} \times 5$.

Example: What is 5 times $\frac{5}{7}$?

$$5 \times \frac{5}{7} = \frac{25}{7} = 3\frac{4}{7} \qquad \qquad Ans.$$

Explanation. Multiplying the whole number 5 by the numerator of the fraction 5 gives the product 25, and

dividing this by the denominator 7 to reduce the improper fraction to a mixed number gives $3\frac{4}{7}$. Let us test this problem by addition: $\frac{5}{7}+\frac{5}{7}+\frac{5}{7}+\frac{5}{7}+\frac{5}{7}=\frac{25}{7}=3\frac{4}{7}$, which is the same result as found before.

Example. What will be the combined thickness of 4 boards each $\frac{7}{8}''$ thick?

$$\cancel{4}\times\frac{7}{\cancel{8}}=\frac{7}{2}=3\frac{1}{2} \qquad\qquad Ans.$$
$$2$$

Explanation. This is a case in which the whole number will be contained exactly into the denominator, without a remainder. Performing this division, 4 goes into 8 two times. Cross out the 4 and the 8 and write the quotient of this division near the 8. Since 7 is left above the line and 2 is left below the line, the result of the multiplication is $\frac{7}{2}$, which, when simplified, equals $3\frac{1}{2}$. It is suggested that the student test this result by multiplying the whole number by the numerator and dividing this product by the denominator. Which method is easier?

28. The Product of Two Fractions. To multiply one fraction by another, we multiply the numerators together for the new numerator and multiply the denominators together for the new denominator.

Example. What is the product of $\frac{3}{4}$ and $\frac{3}{8}$?

$$\frac{3}{4}\times\frac{3}{8}=\frac{9}{32} \qquad\qquad Ans.$$

Explanation. Multiplying the numerators, 3×3 equals 9, which is the numerator of the product. Multiplying the denominators, 4×8 equals 32, which is the denominator of the product. The result is, therefore, $\frac{9}{32}$.

29. " Of " means " Times." The word *of* is often seen in problems in fractions. For example, the statement some-

times appears like this: What is $\frac{3}{4}$ of $5''$? In such a case we work the problem by multiplying, so we say that *of* means *times*. You can see that this is so by taking a piece of wood $5''$ long and cutting it into four equal parts and then taking three of these parts. These three parts will be $\frac{3}{4}$ of $5''$, and by actual measurement, they will be $3\frac{3}{4}''$ long. Thus we know that $\frac{3}{4}$ of $5'' = 3\frac{3}{4}''$. Let us now see what $\frac{3}{4}$ times $5''$ is. $\frac{3}{4} \times 5'' = \frac{15}{4} = 3\frac{3}{4}''$, which is the same value that we found
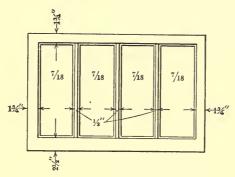

Fig. 15.—Four-light Window Sash.

before. Therefore, we see that the word *of* when used thus signifies multiplication.

30. Multiplying a Group of Whole Numbers and Fractions. If we wish to multiply any number of whole numbers and fractions together, we multiply the whole numbers and the numerators together for the numerator of the product and multiply the denominators together for the denominator of the product. Reduce the result to its simplest form.

Example. What is the product of the following: $2 \times \frac{1}{4} \times 5 \times \frac{3}{8}$?

$$2 \times \frac{1}{4} \times 5 \times \frac{3}{8} = \frac{2 \times 1 \times 5 \times 3}{4 \times 8} = \frac{30}{32} = \frac{15}{16} \qquad Ans.$$

Explanation: The product of the whole numbers and the numerators is 30 and the product of the denominators is 32. This gives $\frac{30}{32}$, which, when reduced to its lowest terms, equals $\frac{15}{16}$.

31. Multiplying Mixed Numbers. To multiply a mixed number by a fraction or to multiply two mixed numbers together, reduce the mixed numbers to improper fractions and proceed as in multiplying fractions, reducing the product to its lowest terms. Never try to multiply mixed numbers without first reducing them to improper fractions.

Example. Multiply $5\frac{3}{8}$ by $7\frac{1}{2}$.

$$5\frac{3}{8} = \frac{43}{8}$$

$$7\frac{1}{2} = \frac{15}{2}$$

$$\frac{43}{8} \times \frac{15}{2} = \frac{645}{16}$$

Reducing, $\dfrac{645}{16} =$ 16)645(40$\frac{5}{16}$ *Ans.*

$$\underline{64}$$

$$5$$

Explanation. First reducing the mixed numbers to improper fractions, $5\frac{3}{8} = \frac{43}{8}$ and $7\frac{1}{2} = \frac{15}{2}$. We then multiply these just as we did when working with fractions. This gives $\frac{645}{16}$, which may be reduced to the mixed number, $40\frac{5}{16}$.

32. Factors. The factors of a quantity are the numbers which multiplied together will make that quantity. The factors of 4 are 2 and 2 because $2 \times 2 = 4$. If we multiply $2 \times 2 \times 3$ we get 12 and, therefore, the factors of 12 are 2 and 2 and 3. Notice that every factor of a quantity will

exactly divide the quantity; that is, will divide it without a remainder.

33. Cancellation. Now that we know what a factor is

we can work many prob-
lems in the multiplication
of fractions and whole num-
bers by the short method of
cancellation. You remem-
ber from a previous lesson
that we can divide both
the numerator and de-
nominator of a fraction by
the same number without
changing its value. Can-
cellation consists in separat-
ing the numerator and the
denominator of the fraction
into their factors and di-
viding by (canceling out)
their common factors. If
a number of fractions and
whole numbers or mixed
numbers are to be multi-
plied together, the method
of canceling can be used
to great advantage. To
show the advantage of the
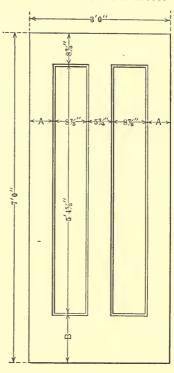method of cancellation we

Fig. 16.—Door.

will work the same example out both ways.

Example. Multiply the following fractions together:

$$\frac{35}{48} \times \frac{34}{21} \times \frac{12}{17}.$$

$$\frac{35 \times 34 \times 12}{48 \times 21 \times 17} = \frac{14{,}280}{17{,}136} \left| 7 = \frac{2040}{2448} \right| 6 = \frac{340}{408} \left| 4 = \frac{85}{102} \right| 17 = \frac{5}{6}$$

<div align="right">Ans.</div>

Explanation. Multiplying all of the numerators together to get the new numerator and all of the denominators to get the new denominator, as we have been previously directed, gives us $\frac{14{,}280}{17{,}136}$. This fraction must be reduced to its lowest terms and we do this by dividing both the numerator and the denominator by the same number. The fraction is so large that we cannot see at first the greatest number that will be contained exactly in both the numerator and the denominator; we will try 7. We write 7 at the right of the vertical line so that we can remember what the divisor is and proceed. The result of the first division is $\frac{2040}{2448}$. We next try 6 as a divisor. The result of this division is $\frac{340}{408}$. Then we divide by 4 and finally by 17 to get the result, $\frac{5}{6}$. We must determine each of these divisors by trial.

Example. Try the same example given above by cancellation.

$$\overset{\displaystyle 5 \quad \overset{\displaystyle 1}{\underset{7}{2}} \quad 1}{\underset{\displaystyle 4 \quad 3 \quad 1}{\frac{35}{48} \times \frac{34}{21} \times \frac{12}{17}}} = \frac{5}{6} \qquad \textit{Ans.}$$

Explanation. Divide 7 into 35 in the numerator and 21 in the denominator. 7 is a factor which is common to both

35 and 21. Cancel the 35 and write 5 above it. Cancel the 21 and write 3 below it. 12 will divide into 12 in the numerator and 48 in the denominator. It goes into 12 once and into 48 four times. 17 will go into 34 in the numerator and 17 in the denominator. It goes into 34 twice and into 17 once. Each time a number is divided we cross it out and write the quotient obtained near it. This is called *canceling*.
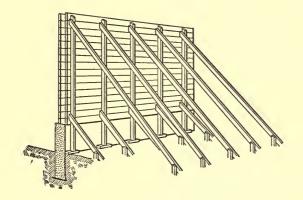

Fig. 17.—Forms for a Concrete Wall.

The numbers now left in the numerator are 5, 2 and 1 and in the denominator 4, 3 and 1. 2 will divide into 2 and 4, leaving 5, 1 and 1 in the numerator and 2, 3 and 1 in the denominator. Now multiply these remaining factors together for the final result, which is $\frac{5}{6}$. It has taken quite a while to tell about this, but in actual work it is much easier and quicker than the method used in the previous example.

SUMMARY OF CHAPTER III

18. **To add mixed numbers,** first add the whole numbers and then the fractions, reducing the result to its simplest form. (Sec. 24.)

19. **To subtract mixed numbers,** subtract the fractions and the whole numbers separately and reduce the result to its simplest form. (Sec. 25.)

20. **To multiply a whole number by a fraction** multiply the number by the numerator and divide the product by the denominator, or if it is possible without a remainder, divide the denominator by the number and reduce the result to its simplest form. (Sec. 27.)

21. **To multiply one fraction by another,** multiply the numerators together for the numerator of the product and the denominators together for the denominator of the product. Reduce the result to its simplest form. (Sec. 28.)

22. **To multiply a group of whole numbers and fractions** together, multiply the numbers and the numerators together for the numerator of the product and multiply the denominators together for the denominator of the product. Simplify the result. (Sec. 30.)

23. **To multiply mixed numbers** together, first reduce them to improper fractions and then proceed as in multiplying fractions. (Sec. 31.)

24. **To multiply by cancellation,** write the numerators as a series of factors and also the denominators. Then cancel out the factors which are common to both the numerators and denominators. Multiply the remaining factors in the denominator for the denominator of the product. Simplify the result, if possible. (Sec. 33.)

PROBLEMS

35. Add the following mixed numbers:

$$6\tfrac{1}{2}'', \; 2\tfrac{9}{16}'', \; 11\tfrac{7}{8}'', \; 5\tfrac{11}{32}''.$$

36. Multiply:

    (a) $2\tfrac{1}{2}\times 8\times \tfrac{1}{16}.$

    (b) $\tfrac{3}{5}\times \tfrac{2}{3}\times 1\tfrac{1}{7}\times \tfrac{7}{8}.$

37. If a carpenter receives 65 cents per hour how much does he receive per day of 8 hours? How much does he get per week of $5\tfrac{1}{2}$ days?

38. If a bricklayer receives $72\tfrac{1}{2}$ cents per hour, how much will he earn per year of 230 days of 8 hours each?

39. What is the cost of $27\tfrac{3}{5}$ sq.ft. of plate glass at $66\tfrac{2}{3}$ cents per square foot?

40. If 20d spikes are worth $4\tfrac{1}{2}$ cents per pound, how much will 23 pounds cost?

41. A turned porch post is shown in Fig. 14. What is the total length of the post? If a 10' post is desired, how long must the distance marked A be if we do not change any other dimensions?

42. A special 4-light window sash is shown in Fig. 15. What is the total height and width of the sash?

43. A $3'\times 7'$ door is shown in Fig. 16. What are the widths of the rails and stiles indicated by letters?

44. Each rise in a certain stair is $7\tfrac{3}{32}''$ high and there are 15 risers. What is the total rise of the stair?

45. The sum of the rise and the tread should about equal 17 in. What should be the width of the tread in Prob. 44 and what is the total run of the stair?

46. Fig. 17 shows an arrangement for the forms for a concrete wall. A contractor finds that he has 32 pieces of form material, each board being $9\tfrac{5}{8}''$ wide actual measurement and 12' long. How high can he build the forms for a 12' section of the wall?

47. A gang of men can mix and place $8\tfrac{3}{10}$ cu. yds. of concrete per hour. How much can they place per day of 8 hours?

48. The size of the Western brick is $8\tfrac{1}{2}''\times 4\tfrac{1}{2}''\times 2\tfrac{1}{2}''$. If the

mortar joint is $\frac{5}{16}''$ thick and the brick are laid flat, how high will a wall be which has 45 courses?

49. A case of shelves is shown in Fig. 18. What is the total height and width of the case?
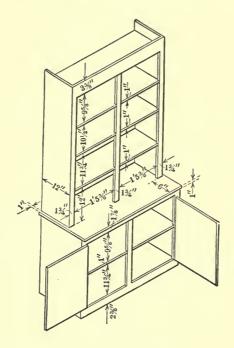

FIG. 18.—A Case of Shelves.

50. The actual face of a $1'' \times 3''$ flooring board is $2\frac{1}{4}''$. A builder has on hand 80 pieces $1'' \times 3''$ flooring 8 ft. long. Will these cover a porch floor 8 ft. wide by 14 ft. long? How many pieces will be left over?

CHAPTER IV

DIVISION OF FRACTIONS AND OF MIXED NUMBERS. COMPOUND FRACTIONS

34. Division of Fractions. If we were required to divide anything by two we would separate it into two equal parts. One of these parts would be one-half of the whole. Suppose that we wish to divide $\frac{3}{4}$ by 2, we would write it thus: (a) $\frac{3}{4} \div 2 = \frac{3}{8}$ or, in other words, one-half of three-quarters is three-eighths. But *of* means the same thing as *times* as explained before. We could write this same operation thus: (b) $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$. Thus we see that the first expression and the second are equivalent. You will notice that the whole number 2 may be written $\frac{2}{1}$, as previously explained, since 2 divided by 1 equals 2. Now let us write these two expressions directly under one another and see what conclusion we may draw.

$$(a) \quad \frac{3}{4} \div \frac{2}{1} = \frac{3}{8}$$
$$(b) \quad \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

Do you see that the first looks like the second except that the division sign in (a) is changed to the multiplication sign in (b) and the second fraction in (b) is the second fraction in (a) only it is turned upside down ?

We can, therefore, write the following rule for the division of fractions: To divide one fraction by another, invert (turn upside down) the divisor (the second fraction) and proceed as in multiplication.

Example. A man left $\frac{2}{3}$ of his land to his children, and $\frac{1}{5}$ of this to his eldest son. What fraction of the land did his eldest son get?

$$\frac{2}{3} \div 5 = \frac{2}{3} \times \frac{1}{5} = \frac{2}{15} \qquad Ans.$$

Explanation. The estate is first divided into three equal parts. Each of these is then divided into five equal parts
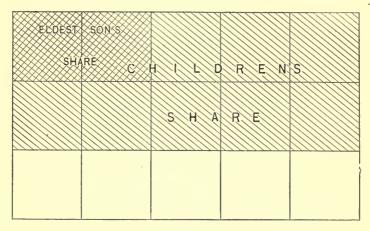

Fig. 19.

to determine the son's share. This division of the land is shown in Fig. 19. If we divide $\frac{2}{3}$ by 5 this would be equivalent to multiplying $\frac{2}{3}$ by $\frac{1}{5}$ and the result of this multiplication is $\frac{2}{15}$, which is the fraction of the original property which the son got.

Example. Divide $\frac{11}{64}$ by $\frac{3}{32}$.

$$\frac{11}{64} \div \frac{3}{32} = \frac{11}{\underset{2}{64}} \times \frac{\overset{1}{32}}{3} = \frac{11}{6} = 1\frac{5}{6} \qquad Ans.$$

Explanation. In this example we follow the rule exactly. Notice also that we can apply the method of cancellation. Do not fail to take advantage of any method by which you can shorten your work. $\frac{11}{64} \div \frac{3}{32}$ is the same as $\frac{11}{64} \times \frac{32}{3}$.

35. Compound Fractions. Division is indicated by a fraction. If we divide one fraction by another, we may express the operation as a compound fraction. A compound fraction has a fraction for a numerator and a fraction for a denominator.

Example. Simplify $\dfrac{\frac{11}{64}}{\frac{3}{32}}$.

$$\frac{\frac{11}{64}}{\frac{3}{32}} = \frac{11}{\underset{2}{\cancel{64}}} \times \frac{\cancel{32}}{3} = \frac{11}{6} = 1\tfrac{5}{6} \qquad Ans.$$

Explanation. This is the same example as the one just preceding but the division is first expressed as a compound fraction. The expression $\dfrac{\frac{11}{64}}{\frac{3}{32}}$ means that $\frac{11}{64}$ is to be divided by $\frac{3}{32}$.

36. Dividing a Whole Number by a Fraction. To divide a whole number by a fraction we write the whole number over the denominator 1, which does not change its value, and proceed as in dividing fractions.

Example. Divide 3 by $\frac{6}{7}$.

$$3 = \tfrac{3}{1}.$$

$$\frac{3}{1} \div \frac{6}{7} = \frac{\cancel{3}}{1} \times \frac{7}{\underset{2}{\cancel{6}}} = \frac{7}{2} = 3\tfrac{1}{2} \qquad Ans.$$

37. Dividing Mixed Numbers. To divide mixed numbers, we first reduce them to improper fractions and then proceed as in dividing fractions.

Example.   Divide $2\frac{1}{2}$ by $6\frac{1}{4}$.

$$2\frac{1}{2} = \frac{5}{2}$$
$$6\frac{1}{4} = \frac{25}{4}.$$

$$\frac{5}{2} \div \frac{25}{4} = \frac{\overset{}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{\underset{5}{\cancel{25}}} = \frac{2}{5} \qquad Ans.$$

38. Solution of Problems.   Problems involving addition, subtraction, multiplication and division or combinations of these operations may be easily solved provided the conditions are properly analyzed.   A problem consists of three parts: the *statement*, the *operation* and the *conclusion* or answer.   The statement of the problem contains the conditions and these must be carefully thought out before an attempt is made to perform the operations.   Be sure that you understand fully what is required by the problems; then you can proceed intelligently with the necessary mathematical operations.   The conclusion or answer should be expressed in its simplest form.

Example.   If 6 men can do the framing for a certain house in 4 days, how long will it take 4 men to do it?

$$6 \times 4 = 24 \text{ days for 1 man to do the work.}$$
$$\tfrac{1}{4} \text{ of } 24 = \ 6 \text{ days for 4 men} \qquad Ans.$$

Explanation.   If 6 men do the work in 4 days, it would take 1 man $6 \times 4 = 24$ days at the same rate of speed.   If 4 men are working together, they can do the work in $\frac{1}{4}$ of this time, or $\frac{1}{4}$ of 24 equals 6 days.

39. Classes of Problems.   Most problems involving fractions fall into one of three classes.

Case 1.   Given the whole to find a part: the whole multiplied by the fraction equals the part.

Case 2.   Given a part to find the whole: the part divided by the fraction equals the whole.

Case 3.   To find what part one number is of another: the part divided by the whole equals the fraction.

Example of Case 1.   A contractor purchases 15,000 lineal feet of 2×4 scantling.   Three-fifths of this order are pieces 18 ft. long and the remainder are 12 ft. long.   How many pieces of each length did he receive?

$\frac{3}{5}$ (the fraction) $\times 15{,}000$ (the whole) $= 9000$ ft. (the part).

$9000 \div 18 = 500$ pieces 18 ft. long   *Ans.*

$15{,}000 - 9000 = 6000$ ft.

$6000 \div 12 = 500$ pieces 12 ft. long   *Ans.*

Explanation.   If the whole amount is 15,000 lineal feet and the part is $\frac{3}{5}$ of the whole, we must find $\frac{3}{5}$ of 15,000 or multiply 15,000 by $\frac{3}{5}$ to get 9000 ft., the required part.   If this 9000 ft. is divided into pieces each 18 ft. long, we must divide by 18 to find the number of pieces.   To find the part which is in 12 ft. lengths, we may subtract the part already found from the whole.   9000 from 15,000 gives 6000 ft. in 12 ft. lengths.   Divide 6000 by 12 to get 500 pieces.

Example of Case 2.   A contractor orders a certain number of brick, two-sevenths of which are to be face brick. He receives 16,000 face brick.   How many brick does he receive in the entire order?   How many common brick does he receive?

$16{,}000$ (the part) $\div \frac{2}{7}$ (the fraction) $= 16{,}000 \times \frac{7}{2} = 56{,}000$

(the whole).

$56{,}000$ brick $- 16{,}000$ face brick $= 40{,}000$ common brick.

$56{,}000$ total; $40{,}000$ common brick; $16{,}000$ face brick.   *Ans.*

Explanation. If 16,000 is $\frac{2}{7}$ of the whole, $\frac{1}{7}$ will be $\frac{1}{2}$ of 16,000 or 8000, and $\frac{7}{7}$ will be 7 times 8000 or 56,000, which is the total number of bricks ordered. This is equivalent to dividing 16,000 by $\frac{2}{7}$. The difference between the whole number of bricks and the number of face bricks will give the number of common bricks.

Example of Case 3. A lumber inspector finds that a board 18 ft. long shows sap for 18 in. of its length and wane for 24 in. What fractional part shows sap and what part wane?

18 ft. $\times$ 12 ins. = 216 ins., the length of the board in inches.

$$\frac{18 \text{ (the part)}}{216 \text{ (the whole)}} = \frac{1}{12} \text{ (the fraction) shows sap.}$$

$$\frac{24 \text{ (the part)}}{216 \text{ (the whole)}} = \frac{1}{9} \text{ (the fraction) shows wane.}$$

$\left.\begin{array}{l}\\\\\end{array}\right\}$ *Ans.*

Explanation. In order that we may not have to deal with mixed numbers it is convenient to reduce the length of the board in feet to inches. Then the part divided by the whole will equal the fraction. 18 divided by 216 (expressed in the form of a fraction and reduced to its lowest terms) gives $\frac{1}{12}$, the fractional part which shows sap. 24 divided by 216 gives $\frac{1}{9}$, the fractional part which shows wane.

SUMMARY OF CHAPTER IV

25. To divide one fraction by another, invert the divisor and proceed as in multiplication.   (Sec. 34 and 35.)

26. To divide a whole number by a fraction, supply the denominator 1 to the whole number and divide as with fractions.   (Sec. 36.)

27. To divide mixed numbers, first reduce them to improper fractions and proceed as in dividing fractions. (Sec. 37.)

28. To find the part when the whole is given, multiply the whole by the fraction.   (Sec. 39.)

29. To find the whole when a part is given, divide the part by the fraction.   (Sec. 39.)

30. To find what part one number is of another, divide the part by the whole to find the fraction.   (Sec. 39.)

PROBLEMS

51. Divide $1\frac{1}{2}$ by $3\frac{3}{8}$.
$2\frac{1}{2} \div 10$.

52. A mortise $\frac{1}{2}''$ wide is to go in the exact center of a piece of $1\frac{3}{4}''$ lumber.   How much wood will be left on each side of it?

53. The distance from the first floor of a dwelling house to the second floor is 9' 10''.   If 16 risers are required in the stair, what will be the rise of each step?

54. A carpenter has a roof surface 22 ft. long and 15 ft. wide to cover with sheathing which measures $5\frac{1}{2}''$ wide.   The boards are spaced 2'' apart.   How many lineal (running) feet of sheathing will he need?

55. How many times may a board 12'' wide be ripped if the strips are to be 1'' wide and the saw cut $\frac{1}{8}''$ wide?   How wide a strip will be left?

56. If shingles are laid $4\frac{1}{2}''$ to the weather, how many courses will be required to cover a roof slope 15' 9'' long?

57. A certain job requires 36 hours to finish it. If a man works at it $1\frac{1}{2}$ hours a day, how many days will it take to complete it?

58. If $\frac{1}{8}$ of a thousand feet of lumber costs $3.00, how much will 1000 feet cost?

59. A pile of lumber contains 12,000 feet. A contractor purchased $\frac{2}{3}$ of it. How many feet did he buy?

60. A man builds three houses which cost him a total of $12,000. The first cost $\frac{1}{3}$ of the whole, the second cost $\frac{2}{5}$ of the whole and the third cost the remainder. What is the cost of each house?

61. A builder buys 240 boards, $\frac{1}{3}$ of which are graded as No. 1 Clear and the remainder are No. 2. How many No. 1 boards and how many No. 2 boards did he receive?

62. A man bought a pile of lumber estimating it to contain 2000 ft. On measuring he found that it contained 1960 ft. What fractional part did he lose?

63. Three-eighths of a thousand feet of lumber costs $24. What is the price of the lumber per thousand?

64. A board weighs 42 pounds green: in drying the weight decreases $\frac{1}{4}$. What does it weigh when dried?

65. In cutting material to lay a certain floor the waste amounts to 50 ft. in every thousand. What fractional part is wasted?

66. A man puts in an eight-hour day. One-third of the day he is employed in laying floor, one-eighth in setting bridge braces and the remainder in finishing. How many hours and minutes does he spend at each kind of work?

67. A fence post is 9 ft. long and $\frac{5}{18}$ of its length is below ground. How many feet are in sight?

68. A man does one-third of a piece of work in $5\frac{1}{2}$ hours. How long will it take him to complete the entire job?

MONEY. ADDITION, SUBTRACTION, MULTIPLICATION
AND DIVISION OF MONEY QUANTITIES. CHECKS.
STATEMENTS. RECEIPTS. ESTIMATES

40. Money. The money unit in the United States is
the *dollar*. The dollar is divided into cents each of which
has a value of one hundredth of the dollar. Fifty cents are
$\frac{50}{100}$ or one-half of one dollar. Twenty-five cents are $\frac{25}{100}$
or a *quarter* of a dollar. The *dime* is one-tenth of a dollar
and the five-cent piece or *nickel* is one-twentieth of a dollar.
To facilitate business transactions, currency is issued in
gold, silver, nickel, bronze and paper money of various
denominations.

The builder should learn to be very accurate in his com-
putations with money and he should be able to keep his
accounts in a neat and orderly manner. If he does this,
he will save himself much trouble and worry. It is not
necessary to know how to keep a set of business books in
order to manage your accounts. Much of the business of
to-day is done without the use of complicated systems of
bookkeeping. A satisfactory record may be made by filing
visible evidences of money transactions such as checks,
receipts, statements, inventories, etc.

41. Writing and Reading Money Sums. When express-
ing money in numerals, the dollar sign ($) is placed before
the number of dollars, a period is placed after this and the
number of cents follows: thus, eight dollars and seventy-

five cents is written $8.75. If cents only are to be written, no figures are written to the left of the period: thus, the sum of eight cents is written $.08. It is also sometimes written 8¢.

42. The Decimal Point. This system gives us our first introduction to decimal fractions. The period mentioned above is the *decimal point*. The first figure to the right of the decimal point indicates the number of dimes or *tenths* of a dollar in the sum. The second figure to the right of the decimal point indicates the number of cents or *hundredths* of a dollar in the sum. This method of writing money quantities greatly simplifies the mathematical operations because we may handle parts or fractions of a dollar in the same manner that we handle whole numbers.

43. Addition. Money quantities are added in the same manner that we add whole numbers. In writing down money quantities for adding, it is necessary to see that all of the periods are directly under one another in a vertical column. If this is done, the cents, dimes and the dollars will be directly under one another. The number of tens of cents may be carried forward and added to the dime column. The number of hundreds of cents will represent dollars and may be added to the dollar column.

Example. Add $3.25, $5.56, $1.00 and $.85.

$$\begin{array}{r} \$3.25 \\ 5.56 \\ 1.00 \\ .85 \\ \hline \$10.66 \end{array}$$

$Ans.$

Explanation. Add the column to the right or the cents column first, $5+0+6+5=16$ cents. This is 6 cents and 1

dime. Put down the 6 and carry the dime to the next column. $1+8+0+5+2=16$ dimes. Put down the 6 and carry the 1 to the dollar column. $1+1+5+3=10$ dollars, which we put down to complete the sum.

44. Subtraction. Subtraction of money is performed as in the subtraction of whole numbers, the only difference being the use of the decimal point separating dollars and cents.

Example. Subtract $6.28 from $9.10.

$$\begin{array}{r} \$9.10 \\ 6.28 \\ \hline \$2.82 \end{array}$$ *Ans.*

Explanation. We set the numbers down with the decimal points under each other just as in addition. We then subtract just as if we were working with whole numbers. In the remainder we place the point directly under the other points to separate dollars and cents.

45. Multiplication. Multiplication of quantities representing money is performed in the same manner as for whole numbers except for the location of the decimal point separating dollars and cents.

Example. If one shipping crate is worth $2.50, how much are 12 crates worth?

$$\begin{array}{r} \$2.50 \\ 12 \\ \hline 500 \\ 2500 \\ \hline \$30.00 \end{array}$$ *Ans.*

Explanation. The multiplication is performed exactly as though we were multiplying whole numbers and the

decimal point is placed in the product by counting two places from the right to mark off the cents.

46. Division. The operation of division of money quantities is the same as for whole numbers. The decimal point is placed in the quotient in the proper position to separate dollars and cents.

Example. A contracting firm bought 5 wheelbarrows for $19.25. What was the price of one?

$$5) \$19.25 (\$3.85 \qquad Ans.$$
$$\underline{15}$$
$$42$$
$$40$$
$$\underline{}$$
$$25$$
$$\underline{25}$$

Explanation. We divide exactly as if we were working with whole numbers and place the decimal point before the last two figures to the right because they represent cents. Another way to locate the point is to place it in the quotient as soon as the number of dollars in the dividend has been divided. For example, in dividing the $19.25 above, we divide 5 into $19 first and get $3. We have then divided all the figures up to the decimal point and we therefore put a point in the quotient after the 3 before we divide again.

47. Bank Accounts. When a man opens a checking account at a bank, he makes a *deposit* sufficient to fully cover all of his expected disbursements for a reasonable period. He is also expected to satisfy the bank that he is conducting a legitimate business. When his application is accepted and his first deposit made, he is given a *pass book* in which his account with the bank is kept. This pass book should

be presented at frequent intervals for a balance. A page
from a contractor's pass book is shown in Fig. 20. On the
left-hand side of the page the abbreviation *Dr.* indicates
that the bank is *debtor* to the depositor on account of cash
deposits. On the right-hand side the abbreviation *Cr.*

Dr. MECHANICS BANK					In Acc't with J.C. Gordon			Cr.	
1914					1914				
June	12	Balance	144	39	June	24	Checks	261	48
"	16	Cash	150	00					
"	24	"	175	00			Balance	207	91
			469	39				469	39
June	24	Balance	207	91	July	1	Checks	191	69
							Balance	16	22
			207	91				207	91
July	1	Balance	16	22	July	10	Checks	140	74
"	1	Cash	140	00					
"	8	"	150	00					

Fig. 20.—Page from a Contractor's Pass Book.

means that the bank credits its account with the depositor
because of checks drawn against his account. The balance
indicates the difference between the amount of cash de-
posited and the sum of the checks drawn against the account.

48. The Bank Check. Fig. 21 shows a bank check
with the stub attached, properly drawn and signed. This
check is an order on the bank to pay the owner of the check

the sum specified and to charge the transaction to the signer of the check. The payee must endorse or sign the check on the back when he presents it to the bank for payment. The check may also be transferred to another party by endorsement. It is the duty of the payee to present the check at the bank for payment as promptly as possible. The stub indicates the number of the check for purposes of identification. It also indicates to whom the check was drawn, the amount and for what purpose it was paid. The
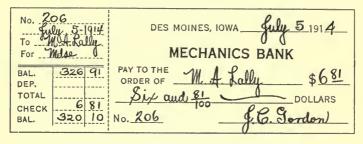

Fig. 21.—Bank Check with Attached Stub.

person who draws the check should also determine the remaining balance to his credit after signing each check so that there will be no danger that he will unknowingly overdraw his account at the bank.

49. The Statement. When a service is performed under verbal or written contract it is customary for the contractor to render an itemized statement on the first of the month immediately following the date on which the work was completed. Such an itemized statement is shown in Fig. 22. This statement serves as a reminder to the person for whom the work was done that he owes the contractor the sum mentioned. Unless there is specific agreement to the con-

DES MOINES, IOWA,

Aug. 30 191 _4_

Mr. _Frank Allan_

1127 University Ave.

To _J. C. Gordon_ Dr.

Terms _Cash in 30 days_

Aug.	15	20 lineal feet 1×3					15
		2–2×8–16' stringers					83
		3–2×10–20' treads				1	56
		1–lb. nails					06
		6 hrs. labor @			80	4	80
						7	40

Received payment
in full
Sept. 13–1914
J. C. Gordon

FIG. 22.—Itemized Statement.

trary, the contractor has a right to expect that the account will be settled promptly.

50. The Receipt. When payment has been made for a service performed it is proper to offer a *receipt*. This should be done even though a receipt is not requested. Fig. 23 shows a common form of receipt for money paid. If the payment is made by check, the endorsement on the back of the check is considered to be sufficient evidence that the money has been paid. It is also often convenient to write

No. 59   No. 59   DES MOINES, IOWA.
$40 25   May 5 191 4
From S. B.   RECEIVED OF S. B. Dodds
Dodds   Forty and 25/100 _____ DOLLARS
For _____   FOR Painting interior and
Service   hanging paper in store
               $40 25   M. A. Lally

FIG. 23.—Form for a Receipt with Attached Stub.

the receipt in the form of a notation on the statement or bill. Fig. 22 shows an example of such a notation.

51. Assets and Liabilities. There is nothing more important in the successful management of the business of the contractor and builder than a frequent and careful estimate of his assets and liabilities. A man's *assets* are his credits and his *liabilities* are his debts. By this means the contractor is able to tell whether he is making money or not. In making a statement of the condition of his business, the contractor should be careful not to include in his assets any items which are really not credits. Unless this point

is carefully watched, a misleading statement may easily be made.

52. Wage Accounts. Every mechanic should know how to keep time and make out a pay roll. Wage accounts should be made out with care and should be properly

TIME——Week Ending		Dec. 13							19/3		
DATE	NAMES	Sun	Mon	Tue	Wed	Thu	Fri.	Sat.	Total Time	Rate per Day	Amount
											$	cts.
	Smith, F. M.		/	/	/	/	/	/	6	3⁵⁰		
	Robinson. David		/	/	/	/	/	/	6	3⁵⁰		
Quit Dec.10	Glover. Chas			/	4/8				2½	3⁵⁰		
	Ninde. Frank		/		/	/	/	4/8	5½	3⁵⁰		
Quit Dec.9	Towson. James		/	3/8					1⅜	3²⁵		
	Penfield. Roy		/	/	/	/	/	/	6	3²⁵		
	Fisher. C. H.		/	/	/	/	/	/	6	3²⁵		
	Moore. Louis		/	/	/	/	/	/	6	3²⁵		
	Morse. Alfred		/	/	/	/	/	/	6	3²⁵		
	Franklin. L. S.		/	/	/	/	/	/	6	2⁵⁰		
	Newton. James		/	/	/	/	/	/	6	2⁵⁰		
	Reece. J.W.		/	/	/	/	/	4/8	5½	2⁵⁰		
	Thompson. Root.					/	/	/	3	1⁵⁰		
	Repass. N. F.					/	/	/	3	2⁵⁰		
	Dolak. T. O.						/	/	2	2⁵⁰		

Fig. 24.—Page of a Time Book.

arranged and preserved. The best way to do this is to use a *time book*. Such a book may be purchased in a form similar to that shown in Fig. 24. In making up pay rolls it is first necessary to find the total amount of time that a workman puts in during a given period. This should be multiplied by his rate to find the amount due him. The

M. A. Lally Residence	14		
22 Windows complete 26 × 28 - 2 lights @ $11 35		249	70
6 Cellar windows 12" × 16" 2 lights, frames etc @ $4 50		27	00
2 Entrance doors complete including hardware @ $26		52	
26 Inside doors complete including hardware @ $13 50			
Base and molding 485 lin. ft. 9" and 2" ¼ oak @ 27¢ 975 lin. ft. 8" and 1½" y.p. @ 14¢			
1 Linen closet @ $40 00			

FIG. 25.—Contractor's Estimate Sheet.

rate is usually given either by the hour or by the day. Adding the amounts due each workman will give the total amount of the pay roll.

53. Building Estimates. The contractor is often called upon to prepare estimates and submit proposals for work about to be done. The utmost care should be taken to make these estimates in a systematic manner and to preserve them. This material is not only valuable for the present purpose, but it has a value for future reference. Fig. 25 shows a contractor's typical estimate sheet.

Summary of Chapter V

31. To add or subtract money quantities, treat them as whole numbers, placing a period in the result to separate dollars and cents.   (Sec. 43 and 44.)

32. To multiply or divide money quantities, proceed as with whole numbers, separating the dollars and cents in the result with a period.   (Sec. 45 and 46.)

PROBLEMS

Note.   When working with dollars and cents always find the answer to the nearest cent; that is, if it is more than half a cent over call it a whole cent and if it is less than half a cent, do not count it.

69. Add the following sums of money: $1.75, $2.38, $9.75, $22, $2.08, $25.00, $11.66.

70. A carpenter agrees to build a wardrobe for $8.50.   The lumber costs $4.00 and the hardware $1.30.   If he spends 16 hours making it, how much does he earn per hour?

71. A contractor agrees to build a barn for $250.   He uses 6000 ft. of lumber at $20 per thousand, 12 squares of roofing at $1.75 per square, and hardware at a cost of $15.   Four men put in 60 hours each at 30¢ per hour.   What is the profit of the contractor?

72. A man sets and cases 4 door frames in a day.   If his rate is $3.25 per day, how much will it cost for him to do this work in a house which has 31 doors?

73. At $3\frac{7}{8}$¢ per pound, how many nails can be bought for $6?

74. If 10d common wire nails are $2.50 per hundred pounds, how much will 35 pounds cost?

75. What will be the price of 28 lineal feet of 14 in. valley tin at $2.75 per 50 feet?

76. The price of one 26×24 check-rail window, glazed double-strength glass, is quoted at $1.84.   How much will 19 such windows cost?

77. A set of front porch steps 6 ft. long with 4 risers and treads will cost $7.50.   How much will steps 7 ft. long cost figuring at the same rate?

78. A porch 7 ft. wide by 16 ft. long is estimated to cost $95. What is the cost per foot of length? What would be the cost of a porch 18 ft. long at the same rate?

79. Determine the amount of money due each man and the total amount of the pay roll shown in Fig. 24.

80. The pass book shown in Fig. 20 gives an open account. Indicate a balance and close the account.

81. The following material and labor was used in making a medicine case 16″×23″ inside measurement: 6 ft. 1×4 jambs, 10¢; 2′ 4″ of stool 1⅛×6, 10¢; 1 sheet of tin for back, 20¢; 4 ft. casing, 12¢; 2 ft. apron, 5¢; 2′ 2″ head casing, 7¢; 3 ft. head mold, 6¢; 1 door, 80¢; 1 beveled plate glass mirror, $1.10; hinges and catch, 25¢; labor, 4 hours at 40¢ per hour. What was the cost of the case?

82. Frank Allan bought the following hardware from the J. C. Kelly Hardware Co., on April 15, 1914: 1 Sliding door set for double doors, $2.25; 5 Mortise latch sets at 55¢ each; 2 Push plates at 30¢ each; 1 Cylinder front door set, $7.25; 8 Inside lock sets at $1.65 each; 28 Flush sash lifts at 3¢; 16 Ball-tipped loose-pin butts 4×4 at 22¢ each; 1 set Parlor door hangers for double doors, $4.00; 1 Double-acting floor hinge, $1.10. Make out a statement for the above material and find the total amount of the bill.

83. What is the total of the estimate shown in Fig. 25?

84. If one concrete finisher at $5.00 per day and five laborers at $2.50 per day are required to lay 700 sq.ft. of cement sidewalk, what will be the labor cost for 100 sq.ft. of walk?

85. To lay 100 sq.ft. of cement sidewalk, the following material is required: 2½ barrels cement at $1.50 per barrel; 1¼ cu.yds. of stone at $1.50 per cu.yd.; ¾ cu.yd. of sand at $1.25 per cubic yard; 2¾ cu.yds. of cinders at 50¢ per cubic yard. What is the total cost of the material? What is the cost of 100 sq.ft. of sidewalk including the labor cost as found in the previous problem? What is the cost per running yard for a walk 4 ft. wide?

CHAPTER VI

DECIMALS. ADDITION, SUBTRACTION, MULTIPLICA-
TION AND DIVISION OF DECIMALS. CHANGING
FROM COMMON FRACTIONS TO DECIMAL FRAC-
TIONS

54. Decimal Fractions. Decimal fractions are different
from common fractions in that they have one of the numbers
10, 100, 1000, etc., for a denominator. Thus $\frac{2}{10}$, $\frac{27}{100}$, $\frac{59}{1000}$,
etc., are really decimal fractions expressed as common frac-
tions. The advantage of using decimal fractions is that
computations are often much simplified.

55. Writing Decimals. Decimals are written in the
same way that we wrote money quantities. In fact, money
quantities are only a special kind of decimals in which we
have only to deal with tenths and hundredths of the dollar:
thus, $1.75 means one and seventy-five hundredths dollars.
In the same manner we may write 1.75 lbs., which means
one and seventy-five hundredths pounds.

In writing decimals we omit the denominator, but we
indicate what the denominator is by means of the decimal
point. The figure which is written after the decimal point
is put down is the numerator of the fraction. For each
figure written to the right of the decimal point we under-
stand that the denominator has 1 and a 0 (cipher): thus,
.5 means that the numerator is 5 and the denominator is 1
and one 0 (cipher) or 10. This decimal expressed as a com-
mon fraction is, therefore, $\frac{5}{10}$. Taking another example,
.19 means that the numerator is 19 and the denominator

is 1 and two ciphers or 100. This fraction, therefore equals $\frac{19}{100}$. It is worth while to state this as a rule: The denominator of a decimal fraction has the unit 1 and as many ciphers following as there are figures to the right of the decimal point.

56. Reading Decimals. The whole number 5280 is read " five thousand two hundred eighty." The word " and " is not used in reading a whole number. A decimal is read like a whole number except that the name of the right-hand number is added: for example, .528 is read " five hundred twenty-eight *thousandths*." You will notice that the denominator of this decimal is 1000. In reading a whole number and a decimal the word " and " is placed between the two: thus, 45.6235 is read " forty-five *and* six thousand two hundred thirty-five ten thousandths." When one person reads numbers containing decimals for another to write, it is customary to say " point " to indicate where the decimal begins; for example, 468.59 would be read " four sixty-eight, *point*, fifty-nine."

57. Significance of Position. If we multiply the unit 1 by 10, we increase its value. If we multiply it by 100, we increase its value still more. If we divide 1 by 10, we decrease its value and if we divide by 100 we decrease its value still more. We express these changes in value by the position which the figure occupies in the expression: thus, 10 has two places of figures; 100 has three places of figures; .1 has one place to the right of the decimal point. The values and places of numbers ordinarily met with are shown in the column below.

1,000,000	Millions
100,000	Hundred thousands
10,000	Ten thousands

1,000	One thousands
100	Hundreds
10	Tens
1	Units
.1	Tenths
.01	Hundredths
.001	Thousandths
.0001	Ten thousandths
.00001	Hundred thousandths
.000001	Millionths

For every place that the unit is moved to the left its value is increased ten times. For every place the unit is moved to the right its value is decreased ten times. It is to be noticed that the location of the decimal point determines the value of the expression. For example, 345 has a value ten times as great as 34.5 because the number of places to the left of the decimal point has been decreased by one. Thus we see that the location of the decimal point is a very important matter.

58. Addition of Decimals. To add numbers containing decimal fractions, it is first necessary to write them so that the decimal points all come directly under one another in a straight vertical line. We then proceed to add as with whole numbers and we place the decimal point in the answer directly under the other decimal points.

Example. Add 692.25, 427.201, .026, 1.5, .3415.

$$
\begin{array}{r}
692.25 \\
427.201 \\
.026 \\
1.5 \\
.3415 \\
\hline
1121.3185
\end{array}
$$
Ans.

Explanation. We must first write the numbers in a vertical column in such a manner that all the decimal points are in the same vertical line. Where the column is not filled out to the right, we are to add as though ciphers were in these places. We place the decimal point in the answer directly under the other decimal points.

59. Subtraction of Decimals. To subtract decimals we first write them so that the decimal points will be directly under each other and then proceed as in subtracting whole numbers. We can " borrow," if necessary, exactly as though working with whole numbers.

Example. Subtract 4.3975 from 10.2.

$$
\begin{array}{ll}
10.2000 & \text{minuend} \\
4.3975 & \text{subtrahend} \\
\hline
5.8025 & \text{difference} \qquad \textit{Ans.}
\end{array}
$$

Explanation. There are four decimal places in the subtrahend. We must, therefore, also have four places in the minuend and we write three ciphers after the 2. We can then borrow and subtract as when dealing with whole numbers. We must not forget to place the decimal point in the result directly under the other decimal points. To check the result we have only to add the subtrahend and difference together and if this sum exactly equals the minuend, we know the result to be correct.

60. Multiplication of Decimals. To multiply decimals, write the numbers so that the figures at the extreme right in both the multiplicand and multiplier are directly under each other. We then proceed as in multiplying whole numbers, forgetting about the decimal point for the moment. When the product has been obtained we point off as many

places from the right as there are decimal places *in both* the multiplicand and multiplier.

Example. Multiply 62.53 by .124.

$$
\begin{array}{r}
62.53 \quad \text{multiplicand} \\
.124 \quad \text{multiplier} \\
\hline
25012 \\
1\ 2506 \\
6\ 253 \\
\hline
7.75372 \\
\end{array}
$$

Ans.

Explanation. We write the two numbers so that the two figures at the extreme right of both the multiplicand and multiplier are in the vertical line. Multiplying as with whole numbers we get the result as shown. To point off the decimal places in the product, we add the number of places in the multiplicand, 2, to the number of places in the multiplier, 3, to get 5. In the answer, therefore, we will point off 5 places beginning at the right and place the decimal point in front of the fifth figure.

61. Division of Decimals. To divide decimal numbers we proceed as with whole numbers, forgetting about the decimal point for the moment. If there are not sufficient figures in the dividend so that the division may be made, we simply add ciphers to the right of the decimal point. This does not change the value of the number. We may carry the operation as far as we please, adding ciphers if necessary. To locate the decimal point in the quotient we point off as many places from the right as there are *more* decimal places in the dividend than in the divisor.

Example. Divide 528.01 by 9.5.

```
Divisor  Dividend  Quotient
 9.5 )   528.01   ( 55.5          Ans.
         475
         ———
         530
         475
         ———
         551
         475
         ———
```

Explanation. This example will give no difficulty until we come to locate the decimal point. There are 2 places in the dividend and 1 place in the divisor. Subtracting the number of places in the divisor from the number in the dividend, 2 minus 1, gives 1 place to point off from the right in the quotient. In the last division there is a remainder, but since this is less than half of the divisor, we neglect it.

Example. Divide 2 by 1.43.

```
1.43 ) 2.0000 ( 1.39               Ans.
       1 43
       ———
        570
        429
       ———
       1410
       1287
       ———
```

Explanation. We must add ciphers to the dividend before we can divide. We place a decimal point after the 2 and add the ciphers to the right. We add as many ciphers as are necessary to get the required number of places in the quotient. If we desired to carry the division farther, we would add more ciphers. To locate the decimal point in the quotient, we subtract the number of places in the divisor from the number in the dividend, $4 - 2 = 2$, and point off two places in the answer.

62. Accuracy of Results. When we were working with common fractions, we could always express a remainder in division by means of a fraction. You will notice that in the above examples in division we might continue to add ciphers to the dividend and carry the division out as far as we please. There will always be a remainder, but the further we carry the operation the more accurate will be the result. For all practical problems we must put a limit to this or the labor involved will be out of all proportion to the accuracy obtained. For all practical problems four places to the right of the decimal point are all that are necessary, and for most cases two or three places are enough. When more than four, or three, or two, as the case may be, occur, we may neglect all of the figures to the right of the fourth, or third, or second, providing the next figure is less than five. If the next figure is more than five, we add one to the last figure that we count and neglect all the others. If the first figure that we neglect happens to be five, we either do or do not add one as our judgment dictates. For example, the number 52.345278 may be written to the fourth decimal place as 52.3453−. The −sign following the number means that the real value is something less than the value given. Taking another example, the decimal 0.02534 may be written to the fourth decimal place as 0.0253+. The + sign means that the real value is something more than that given. It is not always necessary to write the + and − signs as has been done here, but it is frequently done if it makes the work clearer.

63. Changing a Decimal Fraction to a Common Fraction. To change a decimal fraction to a common fraction it is only necessary to supply the denominator and to reduce the resulting fraction to its lowest terms.

Example. Reduce .625″ to a common fraction.

$$.625'' = \frac{625}{1000} \left| 5 = \frac{125}{200} \right| 5 = \frac{25}{40} \left| 5 = \tfrac{5}{8}'' \right. \qquad Ans.$$

Explanation. We first supply the denominator, which will be 1, with as many ciphers following as there are decimal places to the right of the decimal point, or 1000. Reducing this fraction to its lowest terms by dividing by 5 three times, gives the result $\tfrac{5}{8}$.

64. Changing a Common Fraction to a Decimal Fraction. To change a common fraction to a decimal fraction, place a decimal point after the numerator and add as many ciphers after it as are needed, usually four or less, depending upon the number of decimal places desired in the quotient. Then divide the numerator by the denominator and point off as many places in the quotient as there are ciphers added in the dividend.

It is not possible to reduce every common fraction to an exactly equivalent decimal fraction. Fractions having only the prime numbers 2 and 5 in the denominator may be reduced to an exactly equivalent decimal. However, we may reduce any common fraction with sufficient exactness for all practical purposes.

Example. Change $\tfrac{7}{16}''$ to a decimal fraction.

$$\underline{16\,)\ 7.0000}\ (\ \underline{.4375} \qquad Ans.$$
$$\begin{array}{r} 6\,4 \\ \hline 60 \\ 48 \\ \hline 120 \\ 112 \\ \hline 80 \\ 80 \\ \hline \end{array}$$

Table of Decimals of an Inch for Each $\frac{1}{64}$th

$\frac{1}{32}$ds.	$\frac{1}{64}$ths.	Decimal.	Fraction.	$\frac{1}{32}$ds.	$\frac{1}{64}$ths.	Decimal.	Fraction.
	1	.015625			33	.515625	
1	2	.03125		17	34	.53125	
	3	.046875			35	.546875	
2	4	.0625	$\frac{1}{16}$	18	36	.5625	$\frac{9}{16}$
	5	.078125			37	.578125	
3	6	.09375		19	38	.59375	
	7	.109375			39	.609375	
4	8	.125	$\frac{1}{8}$	20	40	.625	$\frac{5}{8}$
	9	.140625			41	.640625	
5	10	.15625		21	42	.65625	
	11	.171875			43	.671875	
6	12	.1875	$\frac{3}{16}$	22	44	.6875	$\frac{11}{16}$
	13	.203125			45	.703125	
7	14	.21875		23	46	.71875	
	15	.234375			47	.734375	
8	16	.25	$\frac{1}{4}$	24	48	.75	$\frac{3}{4}$
	17	.265625			49	.765625	
9	18	.28125		25	50	.78125	
	19	.296875			51	.796875	
10	20	.3125	$\frac{5}{16}$	26	52		$\frac{13}{16}$
	21	.328125			53	.828125	
11	22	.34375		27	54	.84375	
	23	.359375			55	.859375	
12	24		$\frac{3}{8}$	28	56		$\frac{7}{8}$
	25	.390625			57	.890625	
13	26	.40625		29	58	.90625	
	27	.421875			59	.921875	
14	28	.4375	$\frac{7}{16}$	30	60		$\frac{15}{16}$
	29	.453125			61	.953125	
15	30	.46875		31	62	.96875	
	31	.484375			63	.984375	
16	32	.5	$\frac{1}{2}$	32	64	1.	1

Explanation. Since the denominator contains only the prime factor 2 it may be reduced to an exactly equivalent decimal. It is found necessary to add four ciphers to the dividend and, therefore, there will be four decimal places in the answer.

65. Decimal Equivalents of the Fractions of an Inch. It is often necessary to find the decimal equivalent of a given fraction of an inch. It is convenient to have a table giving these values. Such a table is shown on page 68, but all of the values are not filled in, for the reason that the student is expected to fill in the missing ones as part of the problem assignment of this lesson.

Summary of Chapter VI

33. **To add decimals,** write the numbers so that the decimal points are in a vertical line, add as with whole numbers and place the decimal point in the result directly under the other decimal points.   (Sec. 58.)

34. **To subtract decimals,** write the minuend and subtrahend so that the decimal points are in a vertical line, subtract as with whole numbers and place the decimal point in the result directly under the other decimal points.   (Sec. 59.)

35. **To multiply decimals,** proceed as though multiplying whole numbers and point off as many places in the product as there are places in both the multiplicand and multiplier.   (Sec. 60.)

36. **To divide decimals,** proceed as though dividing whole numbers, adding ciphers to the right of the decimal point in the dividend if necessary.   Point off as many places in the quotient as the difference of the number of places in the dividend less the number of places in the divisor.   (Sec. 61.)

37. **To change a decimal fraction to a common fraction,** supply the denominator 1, followed by as many ciphers as there are figures to the right of the decimal point, and reduce the resulting common fraction to its lowest terms.   (Sec. 63.)

38. **To change a common fraction to a decimal fraction,** place a decimal point after the numerator and add as many ciphers after it as are needed.   Then divide the numerator by the denominator and point off as many places in the quotient as there are ciphers added in the dividend.   (Sec. 64.)

PROBLEMS

86. Write the following decimals in figures: Sixty-five thousandths, two hundred twenty-five thousandths, fifty-two and two hundredths, five hundred ninety-six and two-tenths, one half of one-thousandth, seven hundred sixty-five and five one-thousandths.

87. Read the following decimals and write them out in words: 622.215, .075, .2865, 1.024, 100.005.

88. Add up the following decimals an⌐ check the result by adding down:

	(a)		(b)
	532.1		5.22
	52.75		.496
	.025		42.1325
	9.		987.
	27.25		.43
	39.5		.025
	.2		5.02

89. Subtract the following:

(a)   .021 from 1
(b)   2.2   from 10.75
(c)   92.25 from 201.2

90. Multiply:

(a)   2.2 by 31.75
(b)   5.25 by 38.2

91. Divide 4.2 by 3.25 and express the result to three decimal places.

92. Divide 9.035 by 13 and express the result to three decimal places.

93. Reduce the following common fractions of an inch to decimal equivalents: $\frac{3}{8}''$, $\frac{13}{16}''$, $\frac{7}{8}''$ and $\frac{15}{16}''$. These are the values which are missing in the table on page 68 and the student should supply these results in that table.

94. Change the following decimals to common fractions and

reduce the result to lowest terms: (*a*) .0221, (*b*) .0638, (*c*) .6862, (*d*) .8255.

95. How many times is the length .0725′ contained into 16.862′?

96. A carpenter agrees to build a fence 120 ft. long for $45.50. The cost of the lumber is 21.8¢ per lineal foot; the labor cost 7.2¢ per lineal foot and the hardware cost ½¢ per lineal foot. What was the actual cost of the fence and what was the profit?

97. California redwood weighs 26.23 pounds per cubic foot. How much will 15 cu.ft. weigh?

98. A circle is 3.1416 times as far around as across it. Find the number of feet around a circle which is 3.1667′ across.

FIG. 26.—Section of a Steel Pipe.

99. One cubic foot of water weighs 62.5 pounds. Find the part of a cubic foot occupied by one pound of water and express the result in a decimal.

100. If it costs $125 per day to run a gang and a concrete mixer laying 23 cu.yds. of concrete paving material, what is the cost per cubic yard?

101. A screen used for sifting sand is made out of wire which is .0925″ in diameter. What is the size of the opening between the wires if there are 4 wires to the inch?

102. The external diameter of a steel pipe used for a column is 5.563″. The thickness of the metal is .259″. What is the inside diameter of this pipe? (See Fig. 26.)

CHAPTER VII

THE USE OF RULES. PULLEY SIZES. WIDTH OF BELTS. FORMULAS. SHORT METHODS OF MULTIPLICATION AND DIVISION

66. Rules. The practical carpenter is often obliged to use a rule in working out some of his problems. Such rules are of two kinds; those which have a rational mathematical basis and those which are the result of experience. Both of these kinds are valuable so long as they are true. Some of the rules are exact and others merely give an approximate result. You must be sure that you know whether a rule is reliable or not before you use it. It pays to be on the lookout for good, reliable rules which will give the desired result with but little work.

67. Rule for Pulley Sizes. The mill-man frequently has to find the speed of a saw, grindstone or belt-pulley. The following rule is a good one to use in a case of that kind.

To find the number of revolutions of a driven pulley in a given time, multiply the diameter of the driving pulley by its number of revolutions in the given time and divide by the diameter of the driven pulley.

Example. A pulley on the main shaft 48″ in diameter drives a pulley 26″ in diameter on the counter shaft. The main shaft makes 65 revolutions per minute (r.p.m.). How many r.p.m. does the counter shaft make?

$$
\begin{array}{r}
48'' \\
65 \\
\hline
240 \\
288 \\
\hline
\end{array}
$$

26) 3120 (120 r.p.m.   *Ans.*

$$
\begin{array}{r}
26 \\
\hline
52 \\
52 \\
\hline
00
\end{array}
$$

Explanation. Multiplying the diameter of the driving pulley, 48″, by the r.p.m. of the driving pulley, 65, gives 3120. Dividing this by 26, the diameter of the driven pulley, gives 120 r.p.m. for the speed of this pulley. Notice that we multiply the *two things that belonged to the same pulley together.* If you keep this fact in mind, it will help you to remember the rules.

68. Rule for the Width of Belts. The question sometimes comes up in the mill: What width of belt must be used to drive a certain machine? If you know how many horsepower are required you can use the following rule:

To find the width of belt necessary to transmit a given horsepower, multiply the horse-power by 33,000 *and divide this product by the product of the speed of the belt in feet per minute times* 40, *for a single leather belt.*

Example. A steam engine used to furnish power for a mill is rated at 75 horse-power. The driving pulley is 54″ in diameter and the engine runs at 180 r.p.m. How wide a belt of single thickness of leather will be required?

$$54'' = 4.5 \text{ ft.}$$

$$
\begin{array}{r}
3.1416 \\
4.5 \\
\hline
1\ 57080 \\
12\ 5664 \\
\hline
\end{array}
$$

14.13720 ft. around the pulley.

$$
\begin{array}{r}
14.1372 \text{ ft. around the pulley.} \\
180 \text{ r.p.m.} \\
\hline
1130\ 976 \\
1413\ 72 \\
\hline
\end{array}
$$

2544.6960 ft. per min. speed of belt.

$$\frac{75 \times \overset{825}{\cancel{33000}}}{2544.7 \times \cancel{40}} = \frac{61875}{2544.7} \Big) \overline{61875.0} \ \big(\ 24'' \text{ width of belt required.}$$

$$
\begin{array}{r}
50894 \qquad\qquad\qquad Ans. \\
\hline
109810 \\
101788 \\
\hline
\end{array}
$$

Explanation. With the information given, it is first necessary to find the speed of the belt in feet per minute. Every time the pulley travels once around, the belt will travel the same distance. To find the distance around the pulley we multiply the distance across it in feet by 3.1416. This gives 14.1372 ft. around the pulley. The pulley revolves 180 times in a minute and, therefore, the belt will travel 14.1372 times 180 or 2544.7 ft. per minute. From now on we can follow the rule directly. Multiply the horsepower 75 by 33,000 and divide this by the product of 2544.7

times 40. The result when reduced to its lowest terms gives a 24″ belt.

69. Using Letters in Rules. It is possible to shorten the statement of a rule by using letters instead of words. Usually the first letter of the word is used and the mathematical operations are indicated by using the mathematical signs. It is sometimes hard for a mechanic to understand this arrangement and he immediately gets confused and troubled when he sees these *formulas*, as they are called. To show just how this is done and what it means, let us take the following example:

To find the number of board feet in a piece of lumber multiply the width in feet by the length in feet and this again by its thickness in inches, counting a thickness less than one inch as a full inch.

$$Ft.\ B.M. = W \times L \times t$$

The above rule is stated in two ways: first, in words and second with letters. The abbreviation, " Ft. B.M." means " feet board measure." The sign = is used to indicate that the quantity of Ft. B.M. is equal to the value of the expression to the right of the sign. W stands for the width of the board in feet and L stands for its length in feet. $W \times L \times t$ indicates that the value of W is to be multiplied by the value of L and this again by the value of t. t stands for the thickness of the board in inches. Notice that W and L are capital letters and that t is a small letter. This is a short way, used in this case, to show the difference between values given in feet and those given in inches.

70. How to Use Formulas. To show how formulas may be used we will make use of this one which we have just found: Ft. B.M. $= W \times L \times t$. Usually we will know the width, length and thickness of the board and we will want

to find how many feet B.M. the board contains. In this event, we will write the left side of the formula just as it stands, but in writing the right side we will substitute the values for the width, length and thickness of the board which we know. We will then perform the operations indicated on the right side of the formula and reduce the expression to its lowest terms. The result will be a value

FIG. 27.

for the feet B.M. contained in the board. To illustrate this more fully, let us take a specific example.

Example. How many feet board measure are contained in the board shown in Fig. 27, if $W = 9''$, $L = 14'$, and $t = \frac{7}{8}''$?

$$W = 9'' = \tfrac{9}{12}' = \tfrac{3}{4}'$$
$$L = 14'$$

Then Ft.B.M. $= W \times L \times t$

or, substituting the values of W, L and t as given above:

$$\text{Ft.B.M.} = \tfrac{3}{4} \times 14 \times 1$$

Then $\text{Ft.B.M.} = \dfrac{42}{4} \times 1 = \dfrac{\overset{21}{\cancel{42}}}{\underset{2}{\cancel{4}}} \times 1 = \dfrac{21}{2} = 10\tfrac{1}{2}$

Therefore $\text{Ft.B.M.} = 10\tfrac{1}{2}$ *Ans.*

Explanation. The rule states that W and L must be in feet, so we must change the 9″ to feet. L is already in feet. The rule also says to count the thickness less than one inch as a full inch; therefore $\tfrac{7}{8}″$ must be counted the same as 1″. Now let us substitute these values in the formula. For W we will write $\tfrac{3}{4}′$, for L we will write 14 and for t we will write 1. Then we solve the right-hand side by multiplying $\tfrac{3}{4} \times 14$ and this again by 1. This gives us the result $10\tfrac{1}{2}$. The board, therefore, contains $10\tfrac{1}{2}$ ft. B.M.

71. Short Methods. When making computations in connection with this work, the mechanic wishes, first, to get an accurate result and, second, to get the result quickly. All of this requires practice. If you do a little figuring once a month or so, it will take quite a while to make a small calculation and get it accurate. If, on the other hand, you figure a little every day, you will soon be able to do it more accurately and rapidly.

72. Addition. There is no short cut in addition. You can, however, by following a few hints add very rapidly and accurately. When adding, try to pick out combinations that make 10, such as 4 and 6, 7 and 3, etc. Divide a long column of figures into several parts and add each separately. When adding long columns, put down the total of each column separately and add these together for the final result. To check addition, add again beginning at the other end of the column.

73. Subtraction. To check subtraction add the difference and the subtrahend together to get the minuend.

74. Multiplication. To multiply a number by 10, add one cipher to the right or move the decimal point one place to the right. To multiply by 100 add two ciphers or move the decimal point two places. To multiply by 5 ($\frac{10}{2}$) divide by 2 and multiply by 10. To multiply by 25 ($\frac{100}{4}$) divide by 4 and multiply by 100. To multiply by $6\frac{1}{4}$, $8\frac{1}{3}$, $12\frac{1}{2}$, $16\frac{2}{3}$, $33\frac{1}{3}$, divide by 16, 12, 8, 6 or 3 and multiply by 100. To multiply by 9 (10−1) first multiply by 10 and then subtract the multiplicand from this product. To multiply by 11 (10+1) multiply by 10 and add the multiplicand to the product.

To check multiplication, interchange the multiplicand and multiplier and multiply again.

75. Division. Since division is the reverse of multiplication all hints given for multiplication will work for division if reversed.

To check division, multiply the divisor by the quotient to get the dividend.

SUMMARY OF CHAPTER VII

39. To find the speed of the driven pulley, multiply the diameter of the *driving* pulley by its number of revolutions and divide by the diameter of the *driven* pulley. (Sec. 67.)

40. To find the width of single leather belt necessary to transmit a given horse-power, multiply the horse-power by 33,000 and divide this by the product of the speed of the belt in feet per minute times 40. (Sec. 68.)

41. To find the number of board feet in a given piece of lumber, multiply the width in feet by the length in feet and multiply this product by the thickness in inches. (Sec. 69.)

PROBLEMS

103. The Roman designation M is used for 1000 when speaking of board feet: For example, 5M. B.M. means 5000 ft. board measure. Write a rule similar to Rule 41 to find M.B.M.

104. Write a rule to find the diameter of pulley to use when the diameter and speed of the driving pulley and the speed of the driven pulley are known.

105. A main shaft pulley 18″ in diameter makes 160 r.p.m. If the counter shaft must make 220 r.p.m., what must be the diameter of the pulley on the counter shaft?

106. When using a double leather belt we use the same rule as for the single belt, but we use 60 instead of 40 in the rule. What must be the width of belt for a 75 horse-power engine, having a belt wheel 52″ in diameter making 320 r.p.m.?

107. Add the following table of figures both across the page and down the page. What is the grand total? How long did it take you to get it correctly?

4562	4379	8256	4972	3459
7824	4652	3784	9286	4785
5672	8439	5648	7231	2657
4962	1435	4798	6245	3941
1126	2297	4456	6522	4733

108. Multiply the following without using a pencil:

    (a) 76×50 (b) 79.2×5000
    (c) 256×25 (d) $4.74 \times 33\frac{1}{3}$

109. Write out a set of hints for division similar to those for multiplication given in the lesson.

110. Divide the following without using a pencil:

    (a) $9 \div 16\frac{2}{3}$ (b) $450 \div 6\frac{1}{4}$
    (c) $37.6 \div 250$ (d) $720 \div 8\frac{1}{3}$

111. Multiply 624728 by 645 and check.
112. Divide 68,492 by 356 and check.

PERCENTAGE. DISCOUNTS. PROFIT. INTEREST

76. Percentage. To one who has thoroughly mastered fractions and decimals, the subject of percentage should not present any serious difficulties. The words " per centum" are Latin and mean " by the hundred." The words "per cent" are an abbreviation of " per centum " but they are so common that they are now scarcely recognized as an abbreviation. The sign % is used in place of the words: thus, 10% means 10 per cent.

77. Changing a Fraction to a Per Cent. A per cent may be expressed as a decimal or as a common fraction. In fact, a per cent is a fraction. Thus, 10% is the same thing as .10 or $\frac{10}{100}$ or $\frac{1}{10}$ If this is true we may change any fraction into an equivalent form in per cent. This may be done by reducing the fraction to a decimal, setting the decimal point two places to the right and affixing the per cent sign.

Example. One-fifth of the brick taken from the kiln after a certain burning were soft or salmon brick. What was the per cent of soft brick?

$$\frac{1}{5} = \frac{20}{100} = .20 = 20 \text{ per cent soft brick.} \qquad Ans.$$

Explanation. The soft brick were $\frac{1}{5}$ of the whole number. If we change $\frac{1}{5}$ to a per cent we will get the per cent of soft brick. $\frac{1}{5}$ reduced to a decimal equals .20 and this is the same as 20%.

Notice that the sign % indicates two decimal places. The expression 4.2% means four and two-tenths per cent and this expressed as a decimal fraction would be written .042.

Example. Express the fraction $\frac{7}{8}$ as a per cent.

$$\frac{7}{8} = 8 \overline{)\ 7.000\ }\ (\ .875 = 87.5\% \qquad Ans.$$

$$\begin{array}{r} 6\ 4 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline \end{array}$$

Explanation. First change $\frac{7}{8}$ to a decimal fraction by dividing the numerator by the denominator. This gives .875. To change the decimal to a per cent, move the decimal point two places to the right and add the per cent sign: thus, .875 = 87.5%. The result is read " eighty-seven and one-half per cent " or " eighty-seven and five-tenths per cent."

78. Meaning of Terms. Before we can work problems in percentage we must get a clear idea of the meaning of the terms used. The **base** is the number of which the per cent is taken. The **rate** is the per cent taken, often called **rate per cent.** The **percentage** is the part of the base determined by the rate. The **amount** is the sum of the base and the percentage. The **difference** is the base minus the percentage.

79. Analysis of Problems. Problems involving the use of percentage usually fall into one of the three following classes:

Case 1. What is 10% of 250? The base and rate are here given to find the percentage. Multiply the base by the rate to get the percentage.

$$\text{Percentage} = \text{base} \times \text{rate}.$$

Case 2. 25 is what per cent of 250? The base and percentage are here given to find the rate. Divide the percentage by the base to get the rate.

$$\text{Rate} = \text{percentage} \div \text{base}.$$

Case 3. 25 is 10% of what number? The percentage and rate are here given to find the base. Divide the percentage by the rate to get the base.

$$\text{Base} = \text{percentage} \div \text{rate}.$$

Example of Case 1. In ordering $6\frac{1}{2}$ M.B.M. of siding, the builder agrees with the dealer that not more than 8% of the entire order may be in pieces less than 8' long. How many feet B.M. may be less than 8' long?

8% (rate) × 6500 (base) = .08 × 6500 = 520' (percentage) B.M. *Ans.*

Explanation. It is easier to change the per cent to a decimal before we multiply. We could multiply first and then divide by 100 to get the result. In this case the word *of* in the statement of the problem means the same thing as *times* just as it did in common fractions. .08 times $6\frac{1}{2}$ thousand or 6500 equals 520 ft. B.M.

Example of Case 2. In taking off quantities an estimator finds that of 16,000 sq.ft. of flooring, 5600 sq.ft. are to be quarter-sawed oak. What per cent of the total is the oak?

$$\frac{5600 \text{ (percentage)}}{16,000 \text{ (base)}} = .35 = 35\% \text{ (rate) quartered oak. } Ans.$$

Explanation. 5600 is the percentage and 16,000 is the base. Divide 5600 by 16,000 to get the rate. This gives .35 or 35% of the order to be quartered oak.

Example of Case 3. A furniture maker builds desks in mahogany and oak. The number of mahogany desks turned out of the factory in one year was 22, which was 4% of the total output of the desks. What was the total output of the desks?

22 (percentage) $\div 4\%$ (rate) $= 22 \div .04 = 550$ (base) desks.
                                                                Ans.

Explanation. The 22 mahogany desks represent the percentage and 4% is the rate. It is easier to reduce the 4% to a decimal before dividing. 22 divided by .04 gives 550, which is the total output of desks, including both oak and mahogany.

80. Solution of Problems. The solution of these problems does not appear very difficult when the statement of the problem is simple. It is very important and not always easy to determine which quantity is the base, which the percentage and so on. For example, in the following statement it makes a great deal of difference which quantity is used as the base: One contractor bids $4200 on a certain set of plans. A second contractor bids $4800 on the same plans. The first man's price is 14.3% lower than the second man's, using the lower price as a base, but the second man is only 12.5% higher than the first, using the higher price as a base.

It is also important to determine exactly which quantity is the rate. For example, consider the following state-

ment: When solid rock is crushed, it occupies more space than it did before. The per cent of small open spaces or voids is from 30% to 55%. How much space will one cubic yard of solid rock occupy after it is crushed? In this case the volume of the solid rock is 30% less than or 70% of the total space occupied by the crushed rock. 70% is the rate and 1 cu.yd. is the percentage.

81. List Prices and Discounts. In business transactions many quotations are given as list or catalog prices and are subject to discounts. Discounts are usually given thus: " 60% and 20% and 10% off for cash in thirty days." This does not mean that the total discount is 90%, but it means that a discount of 60% is taken from the list price and then a second discount of 20% is taken from what remains. Finally 10% is taken off this last price for cash in thirty days.

Example. The list price on a certain store door lock is $18 with 40% and 10% discounts and 5% for cash in thirty days. What is the cost of the lock if it is paid for within thirty days?

$18.00	list	$18.00	list	$10.80	1st price
40%		7.20	discount	1.08	discount
$7.2000	discount.	$10.80	1st price.	$ 9.72	2d price
		10%			
		$1.0800	discount		
$9.72	2d price			$9.72	2d price
5% off for cash				.48	discount
.4860	discount for cash.			$9.24	final price
					Ans.

Explanation. To find the final cost of the lock we must find the discounts and subtract them from the price in

consecutive order. The first discount allows 40% off from the list price. 40% of $18.00 is $7.20. To get the first discount price we subtract $7.20 from $18.00 to get $10.80. The next discount is 10% off from this price. 10% of $10.80 is $1.08 and we subtract this from $10.80 to get the second discount price. This is $9.72. Now 5% from this price is allowed for cash in thirty days. 5% of $9.72 is 48 cents and $9.72 less 48 cents gives $9.24 as the final price.

82. Profits. Profit is often expressed as a per cent of the actual cost. If a contractor estimates the actual cost of construction to be a certain sum, he adds a certain per cent of the cost for his profit.

Example. The estimated actual cost of a building is is $4280. If the contractor adds 10% for profit, what price does he bid on the job?

$$\$4280 \times 10\% = \$428$$

Adding the profit: $4280
                      428
                    ――――――
                     $4708 price.   *Ans.*

Explanation. 10% of $4280 is $428. Adding this to the actual cost gives $4708 for the price which the contractor bid.

83. Interest. When one man borrows money from another, he pays interest for the use of the money. This interest is usually reckoned at a certain rate per cent per year. The base in this case is called the **principal.** The only difference between figuring percentage and figuring interest is that in figuring interest time must be taken into account. The interest on a certain principal for one year is a certain sum and for two years it is twice that sum.

Interest is *compounded* by adding the interest to the principal at the end of the year and figuring the interest on this amount for the next year.

Example. A man deposits $500 in the bank at 3% interest. What is the mount of the principal and interest at the end of the year?

$500	principal	$500	principal
.03	rate	15	interest
$15.00	interest	$515	amount. *Ans.*

Explanation. For convenience we express the per cent as a decimal rate. Multiplying the principal, $500, by the rate, .03, gives $15 for the interest for one year. Adding this to the principal gives the amount, or $515.

Example: A contractor borrows $1200 for 6 months at the rate of $7\frac{1}{2}$% per annum. What interest does he pay?

$$
\begin{array}{r}
\$1200 \quad \text{principal} \\
.075 \\
\hline
6\ 000 \\
84\ 00 \\
\hline
2\,)\,\$90.000 \quad \text{interest for one year.} \\
\$45.00 \quad \text{interest for six months.} \quad Ans.
\end{array}
$$

Explanation. Reducing the rate $7\frac{1}{2}$% gives .075 as the rate. Multiplying the principal $1200 by this rate gives $90.00 interest for one year. The money was loaned only for 6 months and the interest for that period will be one-half of the interest for a whole year. One-half of $90 is $45, the interest on $1200 for 6 months at $7\frac{1}{2}$% per annum.

Summary of Chapter VIII

42. The words " per cent " are an abbreviation of " per centum," meaning " by the hundred." The sign % means per cent. (Sec. 76.)

43. **To change a fraction to a per cent,** find its decimal equivalent. Then *move the decimal point two places to the right* and affix the per cent sign. (Sec. 77.)

44. **To find the percentage** when the base and rate are given, multiply the base by the rate. (Sec. 79.)

45. **To find the rate** when the base and the percentage are given, divide the percentage by the base. (Sec. 79.)

46. **To find the base** when the percentage and rate are given, divide the percentage by the rate. (Sec. 79.)

47. **To find the cost price** when a series of discounts are given, find the amount of the discounts consecutively, subtracting each in its turn. The final result will be the cost price. (Sec. 81.)

48. **To figure profit,** multiply the estimated cost by the desired rate and add this allowance for profit to the cost. (Sec. 82.)

49. **To figure interest,** multiply the principal by the rate per annum. If the period stipulated is other than for one year do not fail to take this into account. (Sec. 83.)

PROBLEMS

113. What is 25% of 16? of 48? of 90? of 240?

114. 8 is what per cent of 16? of 40? of 80?

115. 30% off of a number leaves 350. What is the number?

116. A pattern maker allows $\frac{1}{8}''$ per foot for shrinkage. What per cent does he allow?

117. A contractor figures that 1890 ft. B.M. will be required for the studding in a house. If he add 25% for waste, what B.M. must he order?

118. In receiving an order for siding, a contractor gets 650 ft. B.M. in 6′ and 8′ lengths out of a total of 5000 ft. B.M. What per cent is this?

119. The actual face of a 4″ floor board is only $3\frac{1}{4}″$. What per cent must be added for matching?

120. Maple flooring " Clear " grade will allow 7% of lengths 2′ to $3\frac{1}{2}′$ according to the grading rules of the Maple Flooring Manufacturers' Association. How many feet B.M. may be of this length in an order for $7\frac{1}{2}$ M.B.M. Clear maple flooring?

121. In laying the floor for a house the contractor allowed 320 ft. B.M. for waste, which was 12% of the total. How many feet B.M. were in the total order?

122. What is the net price of an enameled iron wash bowl if the list price is $18.00 with 30% and 10% off?

123. A contractor figures the actual cost of a job to be $5820. He was awarded the contract at $6550. What per cent of the actual cost was profit?

124. In figuring on a certain job a contractor estimates the actual cost to be $10,490. If he adds 8% for contingencies and 10% of this price for profit, what price will he bid?

125. What is the interest on $500 for 3 years 2 months at $6\frac{1}{2}$% per annum?

126. What will be the total amount of $1200 in 5 years at 5% per year compounded annually?

127. A contractor owns a concrete mixer which cost him $1400. It costs him $15 per day to run the mixer. Figuring money worth 5% and depreciation on the machine at 20%, what will be the total cost of operating the machine per year of 200 days? What will be the cost per day?

CHAPTER IX

RATIO. PROPORTION. CEMENT AND CONCRETE MIXTURES. SLOPES. LEVERS.

84. Ratio. We are constantly comparing things with each other. Among many other things we may compare weight, distance or size. In fact, this is exactly what we do whenever we measure the length of a board. We compare the length of a foot measure with the length of the board and we say that the board is 9 times as long as the foot measure or 9 ft. long. If one plank is 9 ft. long and another is 3 ft. long, we say the first is three times as long as the second or the second is one-third as long as the first. This comparison may also be stated by what we call a *ratio*. That is, we say the *length of the first plank is to the length of the second as* 9 *is to* 3. This is written mathematically, 9 : 3 or $\frac{9}{3}$.

85. A Ratio is a Fraction. When the ratio is written in the form of a fraction, it may be reduced to its lowest terms: thus, $\frac{9}{3} = \frac{3}{1} = 3$, which means that the first board is three times as long as the second. Reversing our comparison we have $\frac{3}{9} = \frac{1}{3}$, which means that the second board is one-third as long as the first. This ratio is said to be the **inverse** of the one just preceding. Thus we see that a ratio may be written like a fraction and, like a fraction, it may be reduced to its lowest terms.

Please notice that *the things to be compared must be of*

the same kind.   For example, it would be absurd to compare feet with bushels.

A ratio is a mathematical comparison of two things of, the same kind.   The two numbers of the ratio are called its **terms.**

Example.   A room is 12′ wide and 18′ long.   What is the ratio of the width to the length?

$$\tfrac{12}{18} = \tfrac{2}{3} = 2 : 3 \qquad\qquad Ans.$$

Explanation.   The comparison of the width to the length will be as 12 is to 18.   Reduced and expressed as a ratio this is 2 : 3.

86. Cement and Concrete Mixtures.   We frequently see the statement that cement mortar is to be mixed in the ratio 1: 3.   This means that 1 part by volume of cement is to be mixed with 3 parts by volume of sand.   A sack of cement weighing 94 pounds contains 1 cu.ft. of cement, very nearly.   Thus for every sack of cement we must have 3 cu.ft of sand.   Likewise in mixing concrete the directions sometimes state that it shall be mixed 1:2½:5 or 1:3:6. Taking the first ratio as an example, this means that the mixture shall be composed of 1 part by volume of cement to 2½ parts by volume of sand to 5 parts by volume of gravel or broken stone.   In order to insure that the mixture shall be uniform and shall completely fulfill the requirements, great care should be taken in measuring and mixing. In Fig. 28 is shown a bottomless measuring box which contains 1 cu.ft. of material for every 3″ of height.   A box like this, which is 12″ high, will contain 4 cu.ft.   Such a box is more accurate and frequently more convenient than the box of an ordinary wheelbarrow for measuring the material.

Example: In making a concrete mixture in the ratio
$1 : 2\frac{1}{2} : 5$ a contractor expects to use 25 sacks of cement.
How many cubic feet of sand and gravel does he require?

25 sacks cement	25 sacks cement
$2\frac{1}{2}$	5
—————	—————
$12\frac{1}{2}$	125 cu.ft. gravel. *Ans.*
50	
—————	
$62\frac{1}{2}$ cu.ft. sand.	*Ans.*

FIG. 28.—Measuring Box.

Explanation. For every sack of cement he will need
$2\frac{1}{2}$ cu.ft. of sand. For 25 sacks he will need $2\frac{1}{2}$ times 25
or $62\frac{1}{2}$ cu.ft. of sand. Also for every sack of cement he will
need 5 cu.ft. of gravel. For 25 sacks he will need 25 times
5 or 125 cu.ft. of gravel.

87. Proportion. If two ratios are equal, the statement of their equality is a **proportion.** The ratio 2 : 3 is equal to the ratio 12 : 18 because the last one mentioned can be reduced to the first: thus, $12 : 18 = \frac{12}{18} = \frac{2}{3} = 2 : 3$. We can, therefore, write these two ratios equal to each other: thus, $2 : 3 = 12 : 18$. Another way of writing this same expression would be $2 : 3 :: 12 : 18$.

To illustrate this further, let us consider the statement: For each 100 sq.ft. of roof surface, 833 shingles are required to lay $4\frac{1}{2}''$ to the weather. Then for 500 sq.ft. of roof surface, 4165 shingles will be required. We can write this statement in the form of a proportion. 100 sq.ft. are to 500 sq.ft. as 833 shingles are to 4165 shingles, or, more briefly, 100 sq.ft. : 500 sq.ft. :: 833 shingles : 4165 shingles. Such a statement as this can, as we have just seen, be expressed mathematically in the form of a proportion.

88. Inverse Proportion. If the four numbers of a proportion are so related that an *increase in one* of the four causes a corresponding *decrease in another*, the proportion is an **inverse proportion.** For example, the following is the statement of an inverse proportion: If it takes 4 men 12 days to do a certain job, 6 men working at the same rate can do it in 8 days. Written as a proportion this would be, 4 men : 6 men :: 8 days : 12 days. Increasing the number of men from 4 to 6 decreases the number of days from 12 to 8.

The first and last terms of a proportion are called the **extremes.** The second and third terms are called the **means.**

89. Solving a Proportion. If we multiply the two means of a proportion together and also the two extremes, the results will be equal. To prove this let us try it with the following proportion:

$$\text{(Extreme)} : \text{(Mean)} :: \text{(Mean)} : \text{(Extreme)}$$
$$2 \quad : \quad 3 \quad :: \quad 12 \quad : \quad 18$$

Product of the means, $3 \times 12 = 36$.

Product of the extremes, $2 \times 18 = 36$.

The two products are equal.

Let us now multiply the two means together and divide the product by one extreme.

$$\text{(Mean)} \times \text{(Mean)} = \text{(Product)}$$
$$3 \quad \times \quad 12 \quad = \quad 36$$
$$\text{(Product)} \div \text{(Extreme)} = \text{(Extreme)}$$
$$36 \quad \div \quad 2 \quad = \quad 18$$

This gives the other extreme.

We will now multiply the two extremes together and divide by one of the means.

$$\text{(Extreme)} \times \text{(Extreme)} = \text{(Product)}$$
$$2 \quad \times \quad 18 \quad = \quad 36$$
$$\text{(Product)} \div \text{(Mean)} = \text{(Mean)}$$
$$36 \quad \div \quad 12 \quad = \quad 3$$

This gives the other mean.

We may write the following statements:

(1) *The product of the two means of any proportion is equal to the product of the two extremes.*

(2) *The product of the two means divided by either extreme gives the other extreme.*

(3) *The product of the two extremes divided by either mean gives the other mean.*

Thus we see if one term of a proportion is unknown it can readily be found by means of these relations.

Example. In the proportion, $25 : x :: 5 : 11$, find the missing term?

$$
\begin{array}{rl}
11 & \text{extreme} \\
25 & \text{extreme} \\
\hline
55 & \\
22 & \\
\hline
\text{Mean} \quad 5\,)\overline{275} & \text{product} \\
55 & \text{mean. } Ans.
\end{array}
$$

Explanation. The missing term is a mean. Therefore, if we multiply the two extremes together and divide the product by the known mean, the result will be the unknown mean, which is indicated in this proportion by the letter x. The product of the extremes, 11 and 25, is 275. Dividing this by the known mean, 5, gives 55 for the unknown term of the proportion.

90. The Statement of a Proportion. Care must be used in making up a proportion from the statement. A good rule to follow is: Put the two things of the same kind in *their proper order* as the first and second term; then write the known thing of the other kind as the third term and indicate the unknown thing as the last term of the proportion.

By the use of the principle of proportion many practical problems may easily be solved.

Example. Joists $2'' \times 12''$, $16''$ on center, are to be set for a floor surface having 960 sq.ft. of area. How many feet B.M. will be required for the joists?

$$16'' = 1\tfrac{1}{3}'$$
$$1\tfrac{1}{3} : 2 :: 960 : x$$

$$\frac{960 \times 2}{1\tfrac{1}{3}} = \frac{\overset{240}{960} \times 2}{\tfrac{4}{3}} = \frac{960 \times 2 \times 3}{4} = 1440 \text{ ft. B.M. joists.} \quad Ans.$$

Explanation. For every foot of floor length there will be a corresponding foot of joist length. For every 16″ or $1\frac{1}{3}'$ of floor width there will be $2 \times 1 = 2$ or 2 board feet of joist. A board foot is one square foot of board 1″ thick. We can, therefore, write this in the form of a proportion. We solve the proportion by multiplying the two means together and

Fig. 29.—Slope of a Rafter.

dividing by the extreme. Expressing this in the form of a fraction and simplifying gives $\dfrac{960 \times 2 \times 3}{4}$ which equals 1440 ft. B.M. of 2×12 joists. The use of the proportion in this problem shortens the work considerably.

91. Slopes. Proportion can frequently be used to advantage in connection with embankments, roof slopes and other similar slopes and grades. *The total horizontal run is to the unit horizontal distance as the total rise is to the rise for the unit run.*

Example. Fig. 29 shows a rafter which rises 6′ 4″ in 16′ of horizontal measurement. How many inches does it rise per foot of horizontal measurement?

$$6' \ 4'' = 76''$$
$$16 : 1 :: 76 : x$$
$$\frac{76 \times 1}{16} = \frac{76}{16} = 4\tfrac{3}{4}'' \qquad\qquad Ans.$$

Explanation. First change the 6′ 4″ to inches because the answer will be in inches. Then writing the proportion, we have: 16 ft. are to 1 ft. as 76 ins. are to the answer. Solving this proportion gives $4\tfrac{3}{4}''$ rise for every horizontal foot.

Example. An earth embankment rises $1\tfrac{1}{2}$ ft. on every foot of level ground. How much will the embankment rise for 18 ft. of level ground?

$$1 : 18 :: 1\tfrac{1}{2} : x$$
$$\frac{18 \times 1\tfrac{1}{2}}{1} = 27 \text{ ft. rise in } 18' \text{ run.} \qquad Ans.$$

Explanation. In this case the proportion can be stated thus: The unit run is to the total run as the rise for the unit run is to the total rise. Writing this in the form of a proportion and solving we get 27 ft. rise in 18 ft. run.

92. Levers. Every mechanic is familiar with the ordinary crow-bar when used as a lever. There are many other familiar examples of the lever. The wheelbarrow is a good example of a lever. The scale beam on which a small weight balances a large weight on the scale platform is another example.

The **lever** is a stiff bar or rod supported or pivoted at a point on which it can move freely. The pivot is called the **fulcrum.** The distance from the center of the weight to the fulcrum is called the **weight arm.** · The distance from the point where the force is applied to the fulcrum is called the **force arm.**

93. Arrangement of Levers. There are three arrange-
ments of levers. The first type is shown in Fig. 30(*a*). The
fulcrum is between the weight and the acting force. Fig.
30(*b*) shows a lever of a second type in which the weight is

Fig. 30 (*a*)

Fig. 30 (*b*)

Fig. 30 (*c*)

between the fulcrum and the force. A lever of a third type
is shown in Fig. 30(*c*). Here the force is applied between the
weight and the fulcrum.

In each of these levers the following proportion holds
good:

Force : Weight :: Weight Arm : Force Arm

Example. A man pushes down on the end of a crow-bar with a force of 140 pounds. The distance from his hand to the fulcrum is 5 ft. How large a weight can he lift at a distance of 1 ft. from the fulcrum?

$$1 : 5 :: 140 : x$$

$$\frac{140 \times 5}{1} = 700 \text{ lbs.} \qquad \qquad \textit{Ans.}$$

Explanation. In writing this proportion we put down the value of the weight arm first. This arrangement brings the answer last. It is not always necessary to do this, but it is convenient. Multiplying the means together and dividing by the extreme gives $\frac{140 \times 5}{1}$ which, when solved, gives 700 lbs. that the man can lift at the end of the bar.

Summary of Chapter IX

50. **To change a ratio to a fraction,** write the first term over the second and reduce this fraction to its lowest terms. (Sec. 85.)

51. **To find the number of cubic feet of sand and gravel** required for any given mixture of concrete, multiply the number of sacks of cement to be used by the second term of the ratio to get the cubic feet of sand required and by

Fig. 31.—Roof Rafter and Strut.

the third term to get the cubic feet of gravel required. (Sec. 86.)

52. **To solve a proportion,** multiply the two means together and divide the product by the known extreme or multiply the two extremes and divide the product by the known mean. The result will be the unknown term or answer of the proportion. (Sec. 89.)

53. **To write the statement of a proportion,** write the two things of the same kind that are known in their proper order as the first and second terms, write the known thing as the third term and indicate the answer as the fourth term. This rule is not rigid but it is convenient. (Sec. 90.)

54. **To solve slopes,** write the proportion: The total horizontal run is to the unit horizontal run as the total rise .is to the rise for the unit run. Then substitute the given values and solve the proportion. (Sec. 91.)

55. **To solve a problem in levers,** write the proportion: Force in pounds is to the weight in pounds as the weight arm in feet or inches is to the force arm in feet or inches. Then substitute the given values and solve the proportion. (Sec. 93.)

FIG. 32.—Wheelbarrow as a Lever.

PROBLEMS

128. A building is three times as long as it is high. What is the ratio of the height to the length?

129. Reduce the ratio $1\frac{1}{2}$: 12 to its lowest terms.

130. Concrete for a certain job is to be mixed 1 : 3 : 5. If 250 sacks of cement are used, how many cubic feet of sand and gravel are required?

131. A drawing is shown to a scale of $1'' = 1'$. What is the ratio of the actual size of the piece to the size of the drawing?

132. If 5 men complete a barn in 18 days, how long will it take 7 men to complete it?

133. If 3 men can make 85 ft. of cement sidewalk 5 ft. wide in a day, how much sidewalk of the same width can 5 men make in one day?   In two days?

134. A contractor knows from past experience that he uses 30 ft. B.M. of 4×4's and 45 ft. B.M. of 2×4's for every 150 ft. B.M. of $1''$ stuff for concrete forms. On a job requiring 22,000 ft. B.M

of 1″ stuff, how many feet B.M. 4×4's and 2×4's will be required?

135. In estimating a certain job a builder finds that there are 2500 sq.ft. of partitions requiring 2×4 studding set 16″ on centers. How many board feet of studding will be required?

136. For a hanging ceiling of 4200 sq.ft. 1×6's set 20″ on center are used for joists.   How many feet B.M. of 1×6's will be required?

Fig. 33.—The Hammer as a Lever.

137. In making a rough estimate a builder remembers that a certain residence containing 32,500 cu.ft. cost $5600.   How much will a residence which contains 47,000 cu.ft. cost at the same rate?

138. The rise of a truck approach at the end of a platform is 3″ for every horizontal foot.   What is the height of the platform if the total horizontal run is 12 ft.?

139. If a roof rafter rises 9′ 6″ in 18′ horizontal run, what must be the length of a vertical strut 11′ from the lower end as shown in diagram in Fig. 31?

140. A wheelbarrow may be considered as a lever of the second

class. This is shown diagramatically in Fig. 32. With what force must a man lift in order to move the wheelbarrow when it is loaded with 350-pounds of material?

141. A claw-hammer is a bent lever as shown in Fig. 33. In drawing a nail a carpenter pulls with a force of 20 lbs. What is the resistance of the nail in pounds?

142. Two men on the end of a bar can push down with a force of 120 lbs. each. If the distance from their hands to the fulcrum is 6 ft. and the distance from the fulcrum to the weight is 18″, how large a weight can they lift?

CHAPTER X

LINEAR MEASURE. AREA. VOLUME. LIQUID MEAS-
URE. DRY MEASURE. WEIGHT. THE CIRCLE.
WEIGHT OF BUILDING MATERIALS. BEARING
POWER OF SOILS. FOOTINGS. CONCRETE MIX-
TURES

94. Weights and Measures. There are two systems of weights and measures in common use. The one most generally used among English-speaking nations is the *British system*. The other system in use is the *metric or decimal system*. The metric system is much more convenient. Although it is used to a considerable extent in some of the metal-working trades, it is doubtful if it will ever be used extensively in the building trades. We will, therefore, confine ourselves to the British system. Tables of values and equivalents of the British system most commonly met with in the building trades are given herewith.

95. Measures of Length. Linear measure is used for comparing lines and distances.

TABLE I

COMMON LINEAR MEASURE

12 inches (in. ″) = 1 foot (ft. ′).
3 feet = 1 yard (yd.) = 36 ins.
5½ yards = 1 rod (rd.) = 16½ ft. = 198 ins.
320 rods = 1 mile (mi.) = 1760 yds. = 5280 ft.

96. Measures of Surface. Surface has two dimensions, length and breadth.

TABLE II

COMMON SURFACE MEASURE

144 square inches (sq.in. □″) = 1 square foot (sq.ft.)
9 square feet = 1 square yard (sq.yd.) = 1296 sq.ins.
$30\frac{1}{4}$ square yards = 1 square rod (sq.rd.) = $272\frac{1}{4}$ sq.ft.
160 square rods = 1 acre (A.) = 4840 sq.yds. = 43,560 sq.ft.
640 acres = 1 square mile (sq.mi.) = 102,400 sq.rds. = 3,097,600 sq. yds. = 27,878,400 sq.ft.

97. Measures of Volume. Volume has three dimensions, length, breadth and thickness.

TABLE III

COMMON CUBIC MEASURE

1728 cubic inches (cu.ins.) = 1 cubic foot (cu.ft.)
27 cubic feet = 1 cubic yard (cu.yd.) = 46,656 cu.ins.

TABLE IV

STONE MEASURE

$24\frac{3}{4}$ cubic feet = 1 perch.

A perch of stone in masonry is $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide and 1 ft. high.

98. Measures of Capacity. Capacity is expressed in units of liquid and dry measure.

TABLE V

COMMON LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.) = 8 pints = 231 cu.ins.
$31\frac{1}{2}$ gallons = 1 barrel (bbl.) = 126 qts. = 252 pts.
2 barrels = 1 hogshead (hhd.) = 63 gals. = 252 qts. = 504 pts.

Dry measure is used for measuring grains, seeds, fruits, lime, etc.

TABLE VI

DRY MEASURE

2 pints = 1 quart.
8 quarts = 1 peck (pk.) = 16 pts.
4 pecks = 1 bushel (bu.) = 32 qts. = 64 pts.

The following table gives the value of some of these units in cubic inches.

TABLE VII

COMPARATIVE VALUES

U. S. liquid measure, 1 gal. = 231 cu.ins.
U. S. liquid measure, 1 qt. = $57\frac{3}{4}$ cu.ins.
U. S. dry measure, $\frac{1}{2}$ pk = $268\frac{4}{5}$ cu.ins.
U. S. dry measure, 1 qt. = $67\frac{1}{5}$ cu.ins.

From the above you will see that a quart liquid measure is smaller than a quart dry measure. The values used in Great Britain are different from the values given here.

99. Measures of Weight. The system used for measuring the weight of all ordinary articles is called **avoirdupois**. Druggists use the apothecary system and jewelers use troy weight. We will use the avoirdupois system.

TABLE VIII

AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 pounds   = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.) = 2000 pounds.
2240 pounds   = 1 long ton.

With these units we are able to make all the measurements commonly used by the carpenter and builder. We

are already familiar with the measures of length; let us now find out how to use measures of surface.

100. Square or Surface Measure. If we cut out a piece of paper in the form of a square one inch on a side, we say that the sheet contains *one square inch* of surface. If we cut out a square, one foot on a side, we say that this contains *one square foot* of surface. The square inch and the square foot are the units of surface measurement. Surface is also often called **area**.

101. Area of the Square and the Rectangle. A **square** surface is one the four sides of which are of equal length and the four angles of which are right angles. *To find the area of a square we multiply the length of one side either in feet or inches by the length of the other side in feet or inches.* The product gives the number of square feet or of square inches in the square. A **rectangle** is a figure similar to the square except that two of its opposite sides are longer than the other two. *To find the area of a rectangle we multiply its length by its width.* It is important to notice that the length and width must be in the same units before you can multiply. For example, feet times feet will give square feet; inches times inches will give square inches. You can change square inches to square feet by dividing by 144 and you can change square feet to square inches by multiplying by 144.

Example. How many square feet does a floor $16' \times 19'$ contain?

$$
\begin{array}{r}
16 \text{ ft.} \\
19 \text{ ft.} \\
\hline
144 \\
16 \\
\hline
304 \text{ sq.ft.}
\end{array}
$$

Ans.

Explanation.   Since the length and width are in feet
the answer will be in square feet.   Multiply the length by
the width to get the area of the rectangle. $16' \times 19' = 304$
sq.ft. in the floor.

Example.   What is the area of a table top $64\frac{1}{2}''$ long
by $28\frac{3}{4}''$ wide?

$$64\frac{1}{2} \times 28\frac{3}{4} = 64.5''$$
$$28.75''$$

$$
\begin{array}{r}
3\ 225 \\
45\ 15 \\
516\ 0 \\
1290 \\
\hline
1854.375 \text{ sq.ins.}
\end{array}
$$

Ans.

Explanation.   For convenience first change the common
fractions to decimals.   Then multiply the length in inches
$64.5''$ by the width $28.75''$ to get the number of square inches
in the area.

102. The Circle.   A circle is a plane figure bounded by
a curved line called a **circumference.**   A line drawn through
the center of the circle and ending on
each side at the circumference is called
a **diameter.**   A line drawn from the
center to the circumference is called a
radius.   The radius is equal to one-
half the diameter.   The meaning of
these terms is further illustrated in
Fig. 34.

Fig. 34.

The diameter of a circle bears a
definite relation to the circumference.
This relation is such that *if we divide the circumference of
any circle by its diameter we will get* $3\frac{1}{7}$ *for an answer.*   If

we divide one quantity by another we have a ratio. This value is, therefore, the *ratio of the circumference to the diameter*. The decimal equivalent of this number is 3.1416. We use this figure so often that we have assigned a special character to name it. This character is the Greek letter pi (pronounced pie) and it is written

$$\pi = 3.1416 = \tfrac{2\,2}{7} = 3\tfrac{1}{7}.$$

It is to be noticed that these values given are not exact, but they are close approximations. We could carry the decimal fraction out to as many places as we pleased, but we would still get a remainder. Four decimal places are all that are necessary for practical purposes. For most purposes the value of $3\tfrac{1}{7}$ or $\tfrac{2\,2}{7}$ is sufficiently accurate. Since π is the ratio of the circumference to the diameter, we can find the circumference when the diameter is given if we multiply the diameter by π.

Example. The diameter of a circle is 56″. What is the length of the circumference?

$$\text{Solution No. 1} \quad \overset{8}{\cancel{56}} \times \frac{22}{\cancel{7}} = 176'' \qquad Ans.$$

Solution No. 2

```
          56
       3.1416
       ------
          336
           56
        2 24
        5 6
      168
      --------
      175.9296''        Ans.
```

Explanation. Here is an example in which the value $\frac{22}{7}$ is very convenient. Using this value and canceling we get the result 176″ in solution No. 1. Compare this to solution No. 2 in which we have multiplied by 3.1416. The result is 175.9296″ or very nearly 176″. This shows that there is no great error in using $\frac{22}{7}$ instead of 3.1416 for the value of π.

Example. The circumference of a circle is 77″. What is its radius?

$$77 \div \frac{22}{7} = \overset{7}{\cancel{77}} \times \frac{7}{\underset{2}{\cancel{22}}} = \frac{49}{2} = 24\frac{1}{2}'' \text{ diam.}$$

$$24\frac{1}{2} \div 2 = 12\frac{1}{4}'' \text{ rad.} \qquad\qquad Ans.$$

Explanation. Divide 77 by $3\frac{1}{7}$ or $\frac{22}{7}$ to get the diameter. The radius is one-half of the diameter, so we divide by 2 to get the radius.

Example. A wooden stave silo is to be built 16′ in diameter. How many lineal feet of $1\frac{1}{4}''$ rod will be necessary for each band, allowing about 9″ for the joint?

$$
\begin{array}{r}
3\,.\,1416 \\
16 \\
\hline
18\ 8496 \\
31\ 416 \\
\hline
\end{array}
$$

$$50\,.\,2656' = 50\frac{1}{4}' \text{ about.}$$
$$9'' = \frac{3}{4}'$$
$$\overline{51'} \qquad\qquad Ans.$$

Explanation. Multiply the diameter 16′ by π to get the circumference of the silo. This gives 50.26,′ which is a little more than $50\frac{1}{4}'$. Adding 9″ or $\frac{3}{4}'$ for the joint gives 51′ required in each band.

103. Volume or Cubic Measure. When we wish to know volume or contents of a cube or rectangular block we find out how many cubic inches or cubic feet it contains. A **cubic inch** is the volume of a cube all of where edges are one inch in length. A **cubic foot** is a cube all of whose edges are one foot in length. *To find the volume of a cube we multiply its length by its breadth and this again by its thickness.* Since all the edges of a cube are equal, this is equivalent to multiplying the length of one side of the cube by itself three times.

Example. How many cubic inches does a cube each of whose edges are 5″ long contain?

$$5'' \times 5'' = 25 \text{ sq.ins.}$$

$$25 \text{ sq.ins.} \times 5'' = 125 \text{ cu.ins.} \qquad Ans.$$

Explanation. If we multiply 5″ times 5″ we get the number of square inches in one face of the cube or 25 sq.ins. Multiplying this again by 5″ gives 125 cu.ins. in the cube.

A rectangular block is similar to a cube except that the length of all the sides is not the same. *To find the volume of a rectangular block, we multiply the length by the width and this again by the height.*

Example. What is the cubic contents of a room 12′ × 16′ × 9′ high?

$$
\begin{array}{r}
12' \\
16' \\
\hline
72 \\
12 \\
\hline
192 \text{ sq.ft.} \\
9' \\
\hline
1728 \text{ cu.ft.}
\end{array}
$$
      Ans.

Explanation.   To find the contents of the room in cubic feet, we multiply the width 12′ by the length 16′ by the height 9′.   This gives 1278 cu.ft. as the contents of the room.

104. Weight of Lumber.   If we know the weight of a cubic foot of lumber we may estimate the weight of a quantity by multiplying the volume in cubic feet by the weight per cubic foot.   The weights of some of the common kinds of lumber are given in the table on page 114.   The weights given are for perfectly dry lumber.   Green timbers weigh from one-fifth to one-half more than the figures given here. Ordinary building timbers, fairly well seasoned, weigh about one-sixth more than these values.

Example.   What will be the weight of a pile of closely stacked Northern pine lumber, air seasoned, which is 6 ft. wide, 18 ft. long and 5 ft. high?

$$
\begin{array}{r}
18' \\
6' \\
\hline
108 \\
5' \\
\hline
540 \\
34.3 \\
\hline
162\ 0 \\
2160 \\
1620 \\
\hline
\end{array}
$$

```
        18′
         6′
        ───
        108.
         5′
        ───
        540    cu.ft.
       34.3    lbs. per cu.ft.
       ─────
      162 0
      2160
      1620
      ──────
6 )18522.0    lbs. dry
     3087     lbs. to add for moisture.
    18522
    ──────
    21609     lbs. weight.              Ans.
```

TABLE IX

WEIGHT OF LUMBER PER CUBIC FOOT

Cypress, American, dry..........	34.3 lbs. per cu.ft.	
Hemlock, perfectly dry..........	25	"
Maple, dry....................	49	"
Oak, white, dry...............	48	"
Oak, red, dry.................	32–45	"
Pine, white, dry..............	25	"
Pine, yellow, Northern..........	34.3	"
Pine, yellow, Southern..........	45	"
Poplar, dry...................	29	"
Spruce, perfectly dry..........	25	"

(Kidder's Pocket Book.)

Explanation. First find the cubic contents of a pile of lumber by multiplying the length by the width by the height. This gives 540 cu.ft. for the volume of the pile. Multiplying this by the weight per cubic foot of Northern pine, which is 34.3 lbs., we get 18,522 lbs. for the weight of the pile if the lumber were perfectly dry. It is only air seasoned and so we must add one-sixth. Dividing by 6 we get 3087 lbs. to add on account of the moisture in the lumber. This gives 21,609 lbs. as the estimated weight of the pile.

105. Weight of Building Material. It is often convenient to know the weight of definite quantities of building materials of various kinds. If we know the weight of a cubic foot of these materials we can find the weight of any quantity just as we did in the case of lumber.

TABLE X

WEIGHT OF VARIOUS BUILDING MATERIALS
PER CUBIC FOOT

	Average weight, pounds per cu.ft.
Brick, best pressed............................	150
Brick, common hard............................	125
Brick, soft inferior..........................	100
Brickwork, pressed brick......................	140
Brickwork, ordinary...........................	112

	Average weight. pounds per cu.ft.
Concrete....................................	150
Granite......................................	170
Limestones and marbles........................	165
Masonry of granite or limestone, well dressed......	165
Masonry of granite rubble......................	154
Masonry of sandstone, well dressed..............	144
Sandstone, fit for building......................	151

(Kidder's Pocket Book.)

Example: How much will an ordinary brick wall 9″ thick by 14′ high by 32′ long weigh?

$$9'' = \tfrac{3}{4}'$$

```
         14'
         32'
         ──
         28
        42
        ───
   4 )448   sq.ft.
      112
        3
      ───
      336   cu.ft. in wall.
      112   weight of brickwork.
      ───
      672
      336
      336
     ─────
    37632   lbs. weight of wall.          Ans.
```

Explanation. We first find the cubic contents of the wall. There are 336 cu.ft. in the wall. Ordinary brickwork, according to the table, weighs 112 lbs. per cubic foot. Hence multiplying the number of cubic feet in the wall by the weight per cubic foot gives, $336 \times 112 = 37{,}632$ lbs. as the weight of the wall.

106. Bearing Power of Soils. The bearing power of various soils depends upon the form of the structure and the

character of the soil. The following table of allowable soil pressures is given by Prof. Ira O. Baker. To be on the safe side the minimum values are recommended. To determine the size of footings we first find the weight of one lineal foot of the wall in tons. Divide this by the allowable pressure per square foot and the result will give the number of square feet per lineal foot required for the footings.

TABLE XI

BEARING POWER OF SOILS

Kind of Material.	Bearing power in tons per sq.ft.	
	Min.	Max.
Rock, the hardest—in thick layers, native bed....	200	
Rock, equal to the best ashlar masonry..........	25	30
Rock, equal to best brick masonry..............	15	20
Rock, equal to poor brick masonry.............	5	10
Clay on thick beds, always dry.................	4	6
Clay on thick beds, moderately dry.............	2	4
Clay, soft....................................	1	2
Gravel and coarse sand, well cemented..........	8	10
Sand, compact and well cemented..............	4	6
Sand, clean, dry..............................	2	4
Quicksand, alluvial soils, etc..................	0.5	1

Example. What size footings will be required for a 12″ concrete wall 46′ high to be placed on clean dry sand?

$$\begin{array}{r} 46 \\ 150 \\ \hline 2300 \\ 46 \\ \hline \end{array}$$ cu.ft. per lineal foot of wall.
wt. per cu.ft. concrete.

2000 lbs.)6900 lbs.

2 tons)3.45 tons

1.725 sq.ft. of footings or,
say, 2 sq.ft. of footings. *Ans.*

Explanation. The wall will contain 46 cu.ft. per foot of length. The weight of concrete is given as 150 lbs. per cubic foot. $46 \times 150 = 6900$ lbs. that the wall weighs per foot of length. Dividing by 2000 gives 3.45 tons. Allowing 2 tons per square foot of bearing surface gives 1.725 sq.ft. of bearing surface required or, to be on the safe side, we will say 2 sq.ft. of bearing surface per lineal foot of wall. The footing should, therefore, be 2 ft. wide.

107. Quantity of Materials Required per Cubic Yard of Concrete. Concrete for building purposes is usually measured in cubic yards. If the number of cubic yards of concrete required is known, the quantities of cement, sand and gravel necessary may be obtained by using the following table. The table is based on one sack being equal to one cubic foot of cement and four sacks to the barrel.

TABLE XII

QUANTITIES OF MATERIALS PER CUBIC YARD OF CONCRETE *

Mixture.	Bbls. Cement.	Cu.yds. Sand.	Cu.yds. Gravel.
$1 : 1\frac{1}{2} : 3$	1.91	.42	.85
$1 : 2 : 3$	1.74	.52	.77
$1 : 2 : 4$	1.51	.45	.89
$1 : 2\frac{1}{2} : 4\frac{1}{2}$	1.31	.48	.87
$1 : 2\frac{1}{2} : 5$	1.24	.46	.92
$1 : 3 : 5$	1.16	.52	.86
$1 : 1$	4.88	.72	
$1 : 1\frac{1}{2}$	3.87	.86	
$1 : 2$	3.21	.95	

* Adapted from a similar table in "Concrete, Plain and Reinforced," by Taylor and Thompson.

Example. A foundation wall is to be made of concrete $1 : 2\frac{1}{2} : 5$ mixture. The wall is 9″ thick, 8′ high and 96′ long. How much material will be required?

$9'' = \frac{3}{4}'$ 96'

             8'

        4)768

         192

          3

     27)576   cu.ft. ($21\frac{1}{3}$ cu.yds. in wall.

       54

       36

       27

       $\frac{9}{27} = \frac{1}{3}$

21.3	21.3	21.3
1.24	.46	.92
852	1 278	426
4 26	8 52	1917
21 3	9.798	19.596
26.412		

$26\frac{1}{2}$ bbls. cement.   10 cu.yds. sand.   20 cu.yds. gravel.
      Ans.   *Ans.*   *Ans.*

Explanation. First find the contents of the wall in cubic feet and divide this by 27 to get the contents in cubic yards. We see from the table that a $1 : 2\frac{1}{2} : 5$ mixture requires 1.24 bbls. of cement for every cubic yard of concrete. Multiply 21.3 cu.yds. by 1.24 to get 26.4112 or, say, $26\frac{1}{2}$ bbls. of cement. In a similar manner it is found that 10 cu.yds. of sand and 20 cu.yds. of gravel are necessary.

Summary of Chapter X

56. To find the area of a square or rectangle, multiply the length by the width.   (Sec. 101.)

57. To find the length of the circumference of a circle when the diameter is given, multiply the diameter by 3.1416. (Sec. 102.)

58. To find the diameter of a circle when the circumference is given, divide the circumference by 3.1416.   (Sec. 102.)

59. To find the volume of a cube or rectangular block, multiply the length by the width and this again by the thickness.   (Sec. 103.)

60. To find the weight of lumber, multiply the number of cubic feet in the closely stacked pile by the weight per cubic foot.   (Sec. 104.)

61. To find the weight of any quantity of building material, multiply the quantity in cubic feet by the weight per cubic foot.   (Sec. 105.)

62. To determine the size of footings, divide the weight of the wall in tons per lineal foot by the bearing power of the soil in tons per square foot.   The result will be the required area of the footing in square feet per lineal foot of wall.   (Sec. 106.)

63. To find the quantities of materials for concrete work, multiply the number of cubic yards of concrete by the value for the required mixture given in the table.   (Sec. 107.)

PROBLEMS

143. How many square feet in a cement sidewalk 6 ft. wide and 180 ft. long?

144. A house is approximately 22′×30′×18′ high.   Allowing 30 sq.yds for gables and dormers, how many square yards of painting surface are there on the house?

145. If a gallon of paint will cover 100 sq.yds. two coats, how many gallons of paint will be required for the above house?

146. If a silo is 18' across, how many feet are there around it?

147. A farmer can measure the distance around his silo with a tape, but he cannot measure across it. He finds that the distance around is $50\frac{1}{4}'$. Tell him the distance across it.

148. How many cubic feet does a bin 6 ft. square and 8 ft. high contain? How many bushels?

149. How many barrels of water will a cistern that is 4 ft. square and 8 ft. high hold?

Fig. 35.—Testing Tank.

150. A testing tank shown in Fig. 35 is built of concrete. It is 6 ft. deep, 14 ft. wide and 25 ft. long. How many barrels of water will it hold?

151. If water weighs $62\frac{1}{2}$ pounds per cubic foot, how much will the water in the above-mentioned tank weigh?

152. How much will a stick of green white oak $12'' \times 12'' \times 10'$ weigh? Add one-fourth to the dry weight to allow for moisture.

153. How much will 500 pieces $2'' \times 4'' \times 16'$ seasoned Northern pine scantlings weigh?

154. If the freight in carload lots from St. Louis to Des Moines is 22¢ per hundred pounds, what will the charges on a load of red oak which measures 8' 4'' wide, 32 ft. long and 6 ft. high when closely piled? Figure 38 pounds per cubic foot.

155. A carload of common hard brick measures 8′ 4″ wide, 5 ft. high and 30 ft. long. What is the weight of the load?

156. A certain concrete chimney contains about $1\frac{3}{4}$ cu.yds. of concrete for every foot of height. How much will a stack 150 ft. high weigh in tons?

157. What must be the size of the footing for the above-mentioned chimney if it is to be placed on gravel and coarse sand well cemented?

158. A chimney for a residence is 35′ high and is made of common hard brick of the dimensions shown in Fig. 36. What must be the size of the footings allowing $\frac{1}{2}$ ton to the square foot?

159. A concrete retaining wall is 18″ wide, 8′ high and 60′ long. A 1 : 3 : 5 mixture is used. How many barrels of cement and cubic yards of sand and gravel will be required?

160. The walls of the testing tank shown in Fig. 35 are 9″

Fig. 36.—Chimney.

thick and the floor is 12″ thick. What will be the weight of the tank when it is full of water? What will be the pressure per square foot of bearing surface?

161. A cement sidewalk is 6′ wide, 180′ long and 4″ in total thickness. The base is a 1 : 3 : 5 mixture and the top coat, which is 1″ thick, is a 1 : $1\frac{1}{2}$ mixture. How much cement, sand and gravel will be required?

CHAPTER XI

POWERS. ROOTS. RIGHT TRIANGLES. AREA OF CIRCLES. CAPACITY OF TANKS AND CISTERNS

108. Powers. It is sometimes convenient and necessary to multiply a number by itself one or more times. This is performed just like any ordinary multiplication, but we have a special name for the product. When a number is multiplied by itself one or more times we call the resulting product the **power** of the number. The number which is used as a factor is call the **base.**

Thus in the expression, $2 \times 2 = 4$, we have raised 2 to second power. The base is 2 and 4 is the power. Again in the expression $2 \times 2 \times 2 = 8$, we have raised 2 to the third power. Instead of writing $2 \times 2 \times 2 = 8$, we may shorten it by writing $2^3 = 8$. The small figure above and to the right of 2 indicates how many times 2 is to be used as a factor.

109. Exponents. The exponent of a power is a small figure placed above and to the right of the base to indicate how many times it is to be used as a factor to get the power. The second power of a number is called the *square* of the number. The third power of a number is called the *cube* of that number. The powers above the third have no special names. They are simply called the *fourth power*, the *eighth power*, etc. The process of finding the powers of numbers is called **involution.**

Example. Find the square of 25.

$$
\begin{array}{r}
25 \\
25 \\
\hline
125 \\
50 \\
\hline
625
\end{array}
$$

$$25^2 = 625 \qquad\qquad Ans.$$

Explanation. To find the square of a number we must use it twice as a factor. 25 times 25 is 625.

Example. Raise 9 to the fifth power.

$$
\begin{array}{r}
9 \\
9 \\
\hline
81 \\
9 \\
\hline
729 \\
9 \\
\hline
6561 \\
9 \\
\hline
59049
\end{array}
$$

$$9^5 = 59049 \qquad\qquad Ans.$$

Explanation. To raise 9 to the fifth power means that we must use 9 as a factor five times.

To raise a number to a given power we must use it as a factor as many times as indicated by the exponent.

110. Powers of Common Fractions. To raise a common fraction to a given power we raise the numerator to the

required power for the new numerator and raise the denominator to the required power for the new denominator.

Example. Find the value of $(\frac{3}{5})^2$.

$$\frac{3}{5} \times \frac{3}{5} = \frac{9}{25} \qquad\qquad Ans.$$

Explanation. To raise a fraction to the second power we multiply it by itself. This is in effect multiplying the two numerators together 3×3, to get 9 for the numerator of the power and multiplying the denominators together, 5×5, to get 25 for the denominator of the power.

111. Roots. Finding the root of a number is just the reverse of finding its power. If we separate 9 into its two factors 3×3, the factor 3 is said to be the square root of 9. In a similar manner, if we separate the number 125 into its three factors $5 \times 5 \times 5$, the factor 5 is said to be the cube root of 125.

The **square root** of a number is one of the two equal factors into which the number may be separated. The **cube root** is one of the three equal factors into which a number may be separated. The **fourth root** is one of the four equal factors, and so on for the higher roots. To indicate that a root is to be taken of a number we use a sign $\sqrt{}$ called the **radical sign.** A small figure placed in the opening of the sign indicates what root is to be extracted. This figure is called the **index** of the root. For example $\sqrt[3]{27}$ indicates the cube root of 27. The small figure 3 is the index of the root. When the square root is intended the index figure is omitted, since it is understood. The operation of finding the root of a given quantity is called **evolution.**

Roots are of two kinds. Those which may be extracted without a remainder are called *perfect roots.* Those which have a remainder are *imperfect roots.* It is impossible to

find the exact value of an imperfect root, but it may be found correctly to any required number of decimal places. Three decimal places are usually sufficient.

112. Square Root. If we square the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, the squares of these numbers will be 1, 4, 9, 16, 25, 36, 49, 64 and 81. Thus 1 is the square root of 1, 2 is the square root of 4, 3 is the square root of 9, 4 of 16 and so on. It is worth while to remember these figures and to be able to recognize them when you see them. The square root of numbers above one hundred are not so easy to determine. To do this we must go through a somewhat complicated process.

Example. Find $\sqrt{119025}$.

$$\sqrt{11'90'25}\ (\ 345 \qquad \textit{Ans.}$$

$$\begin{array}{r|l}
 & 9 \\
64 & 290 \\
 & 256 \\
685 & 3425 \\
. & 3425 \\
\end{array}$$

Explanation. First point off the number into *periods* of two figures each beginning at the right and placing a mark between them.

The next step is to find the largest perfect square which ~~is equal to or less than the first period at the~~ ~~right~~. This *typo err* may readily be determined by inspection. In this case the perfect square is 9 and the root of the square is 3. Put the 9 under the first period, 11, and put its square root, 3, to the right as the first figure in the root or answer. Now subtract 9 from 11 to get 2. Bring down the next period, 90, and write it with this remainder to get 290.

Multiply the first figure of the root, which is 3, by 2 and write the product, 6, at the left of 290 as a trial divisor.

Divide the first two figures of the number 290, which are 29, by this 6 and write the result as the next figure of the root. Six will go into 29 four times and so we write 4 as the second figure of the root to the right of the figure 6 to get 64 as the first divisor.

Multiply the trial divisor, 64, by 4, the second figure of the root, and subtract the product, 256, from 290. The remainder is 34. Bring down the next period to get 3425.

Multiply the first two figures of the root as found, 34, by 2 and write the product, 68, to the left of the number 3425. Using the first three figures, 342, find how many times 68, the second trial divisor, is contained into 342. It is contained 5 times. Write 5 as the third figure in the root. Also write 5 to the right of the 68 to get 685 as the second divisor. 685 is contained into 3425 five times without a remainder. The square root of the number 119,025, is, therefore, 345.

The process of finding the square root of a number which is not a perfect square is similar to that just described. The root of such a number must include a decimal. A decimal point and ciphers must, therefore, be written to the right of the number. We add as many pairs of ciphers as there are to be decimal places in the root.

Example. Find $\sqrt{596}$.

$$\sqrt{5'96.00'00'00} \ (\ 24.413 \qquad Ans.$$

```
                  4
        44     |196
               |176
        484      |2000
                 |1936
        4881       |6400
                   |4881
        48823        |151900
                     |146469
```

Explanation. This is not a perfect square; that is to say there is no number which, multiplied by itself, will give 596.

To extract the root we proceed to point off the number into periods as before explained. Since the root is to contain a decimal we must place a decimal point at the right of the number and add as many pairs of ciphers as there are to be decimal places in the root. We will carry the result to three decimal places and, therefore, we will add three pairs of ciphers to the right of the decimal point. In all other respects the solution is similar to the example given above.

113. Directions for Extracting Square Root. Begin at the decimal point and separate the number into periods of two figures each, going both to the right and to the left of the decimal point. If there is no decimal point begin with the figure farthest to the right. Place a mark between these periods.

Find the largest whole number whose square will be contained in the first left-hand period. Write this number as the first figure of the root. Subtract the square of this number from the first period and annex the second period to the remainder thus found.

Multiply the part of the root already found by 2, and write the result at the left of the first remainder. This gives the first trial divisor. Divide that portion of the dividend except the last figure by the trial divisor, and write the result as the second figure of the root. Annex this figure to the right of the trial divisor to get the first divisor. Perform the division and obtain the remainder, which together with the next period forms the next dividend.

Repeat the process just described above until the last period of the number is used and to the last figure of the

root obtained. If more decimal places are required in the root add as many pairs of ciphers as there are decimal places desired. If the last period of a decimal should contain but one figure, add one cipher to complete the period.

If at any time the trial divisor will not be contained in the dividend, write a cipher as the next figure of the root, bringing down another period and proceed.

There should be as many decimal places in the root as there are periods to the right of the decimal point in the number of which the root is to be extracted.

114. Proof of Square Root. To prove a root multiply it by itself to get the square. If the number is not a perfect square, the proof will not give the exact square, but it will be very close.

Example. Prove the root in the last example given above.

```
           24.413
           24.413
          ───────
            73239
           24413
         9 7652
        97 652
       488 26
       ─────────
       595.994569          Ans.
```

Explanation. Multiplying the root by itself to get the square we get very nearly 596. Since this is not a perfect square the proof will never be exact.

115. Cube Root. To find the cube root of a number means to find a factor which when used three times will give the number. Thus 2 is the cube root of 8 because

$2 \times 2 \times 2 = 8$.   The cube root of 27 is 3, of 64 is 4, of 125 is 5, etc.

To find the cube root of larger numbers we may perform an operation similar to the one for square root. This is somewhat more complicated than the method of finding the square root. Instead of working it out we usually make use of a table of cubes or cube roots or solve the problem by means of logarithms. The carpenter has little occasion to find the cube root and for that reason the process is omitted from these lessons.

116. Practical Applications.   The practical applications of the processes of involution and evolution which occur in the daily work of the carpenter have to do mostly with the properties of regular geometric figures.

117. The Side of a Square.   If the area of a square is given we can find the length of the side by extracting the square root of the area. This is self-evident, because we find the area by squaring the length of the side and to find the side we would naturally expect to extract the square root of the area.

Example.   A builder has sufficient material to lay 1250 sq.ft. of floor. What is the length of the side of the largest square that can be covered?

$$\sqrt{12'\ 50} \, (\, 35.3 \text{ ft.} \qquad Ans.$$

$$
\begin{array}{r|l}
      & 9 \\ \hline
   65 & \overline{350} \\
      & 325 \\ \hline
  703 & \overline{2500} \\
      & 2109
\end{array}
$$

Explanation.   Extracting the square root of 1250 gives 35.3 approximately. This is the length in feet of the side of the square which can be covered by the material.

118. The Right Triangle. A triangle is a figure which has three straight sides and three angles. A **right triangle** is a triangle one of whose angles is a right angle. Fig. 37 represents a right triangle. The side marked a is called the **altitude**; the side marked b is called the **base** and the side marked c, which is the longest side, is called the **hypotenuse**. To find the length of the hypotenuse when the length of the altitude and base are known, we may square the value of the length of the base and the length of the altitude, add them together and extract the square root of this sum.

FIG. 37.
A Right Triangle.

Example. What is the length of the hypotenuse of a right triangle whose base is 18″ and whose altitude is 12″?

$$
\begin{array}{ccc}
18 & 12 & 144 \\
\times & \times & + \\
18 & 12 & 324 \\
\hline
144 & 24 & 468 \\
18 & 12 & \\
\hline
324 & 144 &
\end{array}
$$

$$\sqrt{4'68'.00'00}(21.63'' \qquad Ans.$$

$$
\begin{array}{r|l}
& 4 \\
41 & 68 \\
& 41 \\
\hline
426 & 2700 \\
& 2556 \\
\hline
4323 & 14400 \\
& 12969
\end{array}
$$

Explanation. We square the value of the base 18″ to get 324 and the value of the altitude, 12″ to get 144. Adding these two together gives 468 for their sum. The square root of 468 is 21.63. The hypotenuse of the triangle is, therefore, 21.63″ long.

We may also find the length of the altitude of a right-angled triangle if we know the length of the hypotenuse and base by turning this rule around. The altitude of a

Fig. 38.

right triangle is equal to the square root of the difference of the squares of the hypotenuse and the base. We may also say that the base is equal to the square root of the difference of the squares of the hypotenuse and the altitude.

Example. In Fig. 38 the length of the rafter from the ridge center to the plate is 14.42 ft. and one-half the span of the roof is 12 ft. What is the vertical distance from the plate to the ridge?

$$\begin{array}{r} 14.42 \\ 14.42 \\ \hline 2884 \\ 5\ 768 \\ 57\ 68 \\ 144\ 2 \\ \hline 207.9364 = 208 \end{array} \qquad \begin{array}{r} 12 \\ \times\ 12 \\ \hline 24 \\ 12 \\ \hline 144 \end{array}$$

$$\begin{array}{r} 208 \\ -\ 144 \\ \hline 64 \end{array}$$

$$\sqrt{64} = 8' \qquad\qquad Ans.$$

Explanation. We first find the square of the hypotenuse, which is 208, very nearly, and then the square of the base, which is 144. The difference between them is 64. The square root of this difference is 8. The required distance is, therefore, 8 ft.

119. The Circle. The processes of involution and evolution are quite common in connection with calculations of the circle. If we wish to find the number of square inches in a circle, for example, we square the radius in inches and multiply this result by π or 3.1416. Also if we wish to find the radius of a circle when the area is given we divide the area by 3.1416 and extract the square root of this quotient.

Example. Find the area of a circle in square inches, the radius of which is 30″.

$$30^2 = 900$$
$$900 \times 3.1416 = 2827.44 \text{ sq.ins.} \qquad Ans.$$

Explanation. Multiplying 30 by itself to get its square gives 900 and multiplying this by π or 3.1416 gives 2827.44 sq.in. in the circle.

Example. What is the radius of a circle whose area is 1017.87 sq. in.?

```
3.1416)1017.8700 ( 324
            942 48

              75 390
              62 832

              12 5580
              12 5664

               √3'24 ( 18''          Ans.
                    1
                   ──
          28       224
                   224
```

Explanation. This problem is just the reverse of the one given above. Divide 1017.87 by π to get 324, nearly. Extract the square root of 324 to get 18'', which is the required radius.

If we work with the diameter instead of the radius we use the value .7854 in place of 3.1416. The radius of a circle is one-half the diameter; therefore the square of the diameter will be four times the square of the radius. This accounts for the use of the value .7854 instead of 3.1416 when using the diameter. $.7854 = \frac{1}{4}$ of 3.1416.

120. Capacities of Circular Tanks and Cisterns. The builder often finds it necessary to determine the number of gallons or barrels that a cistern will hold. This information may usually be found in tables, but it is convenient to know

how to figure it so that if you do not have a table at hand you can figure it out for yourself.

Find the area of the bottom of the tank in square feet and multiply this by the height in feet. This product gives the contents in cubic feet. There are $7\frac{1}{2}$ gallons of water to every cubic foot, so, in order to get the contents in gallons, we multiply the total cubic feet by $7\frac{1}{2}$. There are $31\frac{1}{2}$ gallons to a barrel; therefore, to get the capacity in barrels we divide the total number of gallons by $31\frac{1}{2}$. Multiplying by $7\frac{1}{2}$ and dividing by $31\frac{1}{2}$ is the same as multiplying by .238, because $7\frac{1}{2} \div 31\frac{1}{2} = .238$. Hence to find the capacity in barrels we may multiply the contents in cubic feet by 238.

Example: Find the number of gallons a circular tank will contain if the diameter is 6 ft. and the height is 10 ft.

$$6^2 = 36$$

$$\begin{array}{r}
36 \\
.7854 \\
\hline
144 \\
180 \\
2\ 88 \\
25\ 2 \\
\hline
28.2744 \quad \text{sq.ft.} \\
28.27 \\
10 \\
\hline
282.70 \quad \text{cu.ft.} \\
282.7 \\
7.5 \\
\hline
141\ 35 \\
1978\ 9 \\
\hline
2120.25 = 2120\frac{1}{4} \text{ gallons.} \qquad Ans.
\end{array}$$

Explanation. The square of the diameter 6 is 36 and
the product of this multiplied by .7854 gives 28.27 sq.ft.
in the bottom of the cistern. It is 10 ft. high and 10 times
28.27 gives 282.7 cu.ft. as the contents of the cistern. Mul-
tiplying this by 7.5 gives $2120\frac{1}{4}$ gallons as the capacity of
the cistern.

Example. How many barrels will this tank contain?

$$
\begin{array}{r}
31.5 \,) \,2120.25 \,(\,\underline{67.3} \text{ barrels.} \qquad Ans. \\
1890 \\
\overline{} \\
2302 \\
2205 \\
\overline{} \\
975 \\
945 \\
\overline{}
\end{array}
$$

Explanation. We have just figured the number of
gallons the tank will contain. There are $31\frac{1}{2}$ gallons to
the barrel. If we divide the number of gallons 2120.25
by 31.5 the result, 67.3, will give the answer in barrels.

Example. How many barrels will a cistern hold the
cubic contents of which is 282.7 cu.ft.?

$$
\begin{array}{r}
282.7 \quad \text{cu.ft.} \\
.238 \\
\overline{} \\
2\ 2616 \\
8\ 481 \\
56\ 54 \\
\overline{} \\
67.2826 = 67.3 \text{ barrels.} \qquad Ans.
\end{array}
$$

Explanation. This example is used to show that if we
multiply the cubic contents of a cistern expressed in cubic

feet by .238 we will get the same result approximately as to multiply by $7\frac{1}{2}$ and divide by $31\frac{1}{2}$. The value given here, 282.7 cu.ft., is the same as that found in the second example above. Multiplying this by .238 gives 67.2826 barrels, which is very nearly 67.3 barrels as found above.

Summary of Chapter XI

64. To find the power of a number, use it as many times as a factor as is indicated by its exponent: that is, to find the second power use the number twice as a factor; to find the third power use the number three times as a factor. (Sec. 109.)

65. To raise a fraction to a given power, raise the numerator to the required power and also the denominator. (Sec. 110.)

66. To find the square root of a number, follow the explicit directions given in Sec. 113.

67. To prove a square root, multiply the root by itself to get the square. (Sec. 114.)

68. To find the side of a square when its area is given, find the square root of the area. The result will be the length of the side of the square. (Sec. 117.)

69. To find the length of the hypotenuse of a right triangle when the base and the altitude are given, square the base and the altitude and extract the square root of their sum. (Sec. 118.)

70. To find the length of the altitude of a right triangle, square both the hypotenuse and the base and extract the square root of their difference. (Sec. 118.)

71. To find the length of the base of a right triangle, square the hypotenuse and altitude and extract the square root of their difference. (Sec. 118.)

72. To find the area of a circle, square the radius and multiply by 3.1416 or square the diameter and multiply by .7854. (Sec. 119.)

73. To find the diameter of a circle when the area is given, divide the area by .7854 and extract the square root

of the quotient. If the radius is wanted, divide by 3.1416 and extract the square root of the quotient. (Sec. 119.)

74. **To determine the contents of a circular tank** in gallons, find the area of the bottom in square feet, multiply by the height of the tank in feet and multiply this by $7\frac{1}{2}$. (Sec. 120.)

75. **To find the capacity of a circular tank in barrels,** multiply the contents in cubic feet by .238. (Sec. 120.)

FIG. 39.—Roof for a Lean-to.

PROBLEMS

162. Find the value of the following:

(a)	$\sqrt{178084}$	422	Ans.
(b)	$\sqrt{198.1369}$	14.076	Ans.
(c)	$\sqrt{.571384}$	.7559	Ans.

163. Find the value of the following:

(a) 14^3	(c) $9^2 \times 3.1416$
(b) 286^2	(d) $(\frac{3}{4})^2$.

164. A person standing requires a space 20″×20″. What are the dimensions of a square platform large enough to hold 75 people, allowing 1 ft. extra all around the edge?

165. What will be the length of a brace cut for 6 ft. run and 6 ft. rise?

166. Fig. 39 shows a roof for a lean-to. The run is 13 ft. and the rise is 6 ft. Find the necessary length of a 2×4 rafter allowing 12″ for overhang and 6″ for trimming. What length of 2×4 would you order?

Fig. 40.—Scissors Roof Truss.

167. What will be the length of a rafter for 12 ft. run and 10 ft. rise allowing about 4″ for trimming and 2 ft. for overhang?

168. Fig. 40 shows a scissors roof truss and the diagram to the right gives the dimensions. What is the length of the main rafter A and what is the length of the strut B?

169. Fig. 41 shows the stringer for a flight of stairs. What is the length of the stringer?

170. Fig. 42 shows a queen-post truss and the diagram to the right gives the dimensions on the center lines. Find the missing dimensions which are indicated by letters.

171. A circular platform has an area of 240½ sq.ft. What is its diameter?

172. What is the area in square inches of a circle two feet in diameter?

173. A circular window opening is 54″ in diameter. How many square feet of glass surface does it contain?

174. What is the capacity of a circular cistern 5 ft. in diameter and 8 ft. deep? Give the answer in barrels.

Fig. 41.—Stringer for a Flight of Stairs.

175. A hot-water tank for an apartment house is said to hold 100 gallons. It is 22″ in diameter and 60″ high. Does it hold 100 gallons as stated? If not, what is its capacity?

Fig. 42.—Queen-post Truss.

176. A circular cistern is 5′ 6″ in diameter and 10′ deep. How large must a square cistern be built of the same height to contain the same amount of water? Why are cisterns usually built round instead of square?

177. A 40-gallon tank is 18″ in diameter. How high must it be?

178. A concrete silo is 16 ft. in diameter and 30 ft. high. What is its capacity in cubic feet?

179. Silage weighs about 40 lbs. per cubic foot. How many tons of silage will the above silo hold?

180. What must be the height of a circular silo to hold 10,000 cu.ft. if its diameter is 18 ft.?

181. What must be the diameter of a drain tile into which one 4″ and one 6″ tile empty? The large tile must have an area equivalent to the two small ones.

CHAPTER XII

LUMBER TERMS AND DIMENSIONS. FLOORING. SHINGLES.

121. Lumber Trade Customs. Our methods of grading and measuring lumber are the result of custom and these customs vary somewhat in different localities. The sizing and grading of lumber is regulated by the various lumber manufacturers' associations. Some of the terms and customs are of such wide usage that they may be explained here.

122. Lumber Terms. The term **timber** is generally applied to sticks more than 4″ in thickness the smaller way, and the term **lumber** is applied to sticks less than 4″ in thickness. A **plank** is a piece from $1\frac{1}{2}$″ to 4″ thick; while a **board** is a piece less than $1\frac{1}{2}$″ in thickness. A **scantling** is a piece of lumber 4″×6″ in size or less. The term is most frequently applied to a 2″×4″ stick, though it may refer to a 4″×4″ or 2″×6″ stick equally well.

Rough stock means lumber that is sawn so as to admit of dressing but that is not dressed. **Dressed stock** means lumber that has been planed or surfaced on one or more sides. S. 1 S is an abbreviation for " surfaced on one side." S. 2 S. means " surfaced on two sides," S. 1 S. 1 E. means " surfaced on one side and one edge " and S. 4 S. means " surfaced on four sides."

Dimension Lumber is usually dressed on one side and one edge in order to size it. **Finishing lumber** is lumber

intended for interior or exterior finishing.   It is dressed
on one or both sides or as directed.

123. Allowance for Dressing.   In planing or dressing a
board about $\frac{1}{8}''$ is removed for each surface dressed.   Lumber

Fig. 43.—Garage.

which has not been thoroughly dried and seasoned before
being dressed will shrink sufficiently to cause considerable
reduction in the width of a board.   A board which is nom-
inally one inch in thickness which has been dressed on one
or both sides will measure actually about $\frac{13}{16}''$ in thickness.
If lumber is wanted to full dimensions after it is dressed,

it is necessary to specify that the dimensions must be full size after the stock is dressed. Dressed stock is measured **strip count;** that is, at the full size of the rough material used to manufacture the piece.

Fig. 44.—Floor Plan of a Small Cottage.

124. Width of Lumber. Boards run in multiples of one inch in width. Owing to the processes used in manufacture there may be considerable actual variation from the rule. If a board is more than $\frac{1}{4}''$ scant on the 8'' width or of less width it is counted in the next lower width. A board may be $\frac{3}{8}''$ scant on the 9'' or 10'' widths or $\frac{1}{2}''$ scant on the 11''

or 12″ widths or wider. If the boards of any specified nominal width do not meet these requirements they are counted in the next lower width.

FIG. 45.—Plan of Roof.

125. Standard Lengths. Rough stock or finish lumber is usually cut in multiples of two feet in length. Standard lengths for finishing lumber are usually in multiples of one foot.

126. Sizes of Common Boards. Boards of 1″ nominal

size S. 1 S. or S. 2 S. as previously explained, are dressed to
$\frac{13}{16}''$ thick; $1\frac{1}{4}''$ nominal size S. 1 S. or S. 2 S. are dressed
to $1\frac{1}{16}''$; $1\frac{1}{2}''$ nominal size S. 1 S. or S. 2 S. are dressed to
$1\frac{5}{16}''$.

127. Sizes of Dimension Lumber. The sizes of dressed
scantling and dimension lumber are given in the following
table:

FIG. 46.—Forms for Concrete.

TABLE XIII

SIZES OF DIMENSION LUMBER

Nominal Size.	Dressed.	Actual Size.
2×4	S. 1 S. 1 E.	$1\frac{5}{8}'' \times 3\frac{5}{8}''$
2×6	S. 1 S. 1 E.	$1\frac{5}{8}'' \times 5\frac{5}{8}''$
2×8	S. 1 S. 1 E.	$1\frac{5}{8}'' \times 7\frac{1}{2}''$
2×10	S. 1 S. 1 E.	$1\frac{5}{8}'' \times 9\frac{1}{2}''$
2×12	S. 1 S. 1 E.	$1\frac{5}{8}'' \times 11\frac{1}{2}''$

If the lumber is dressed on all four sides, the pieces may
be $\frac{1}{8}''$ less in thickness and width than S. 1 S. 1 E.

128. Sizes of Dressed Finishing Lumber. Finish lum-
ber is dressed to the same thickness that common boards

are. Thus, $1''$ nominal size S. 1 S. or S. 2 S. is dressed to $\frac{13}{16}''$ in thickness; $1\frac{1}{4}''$ S. 1 S. or S. 2 S. is dressed to $1\frac{1}{16}''$ in thickness; $1\frac{1}{2}''$ S. 1 S. or S. 2 S. is dressed to $1\frac{5}{16}''$ in thickness. Stock which is nominally $4''$ in width is dressed to $3\frac{1}{2}''$ when finished. Other widths are also dressed to $\frac{1}{2}''$ less than nominal widths.

129. Flooring. To allow for the tongue and groove matching $1''\times3''$, $1''\times4''$ and $1''\times6''$ flooring will run $\frac{13}{16}''$ thick and show $2\frac{1}{4}''$, $3\frac{1}{4}''$ and $5\frac{1}{4}''$ face.

Fig. 47.—Cupboard.

130. Sizes of Framing Lumber. Joists are reduced $\frac{1}{4}''$ on the side and $\frac{1}{2}''$ on the edge if surfaced on one side and one edge when green and unseasoned. If they are surfaced on four sides, they are reduced $\frac{1}{4}''$ for each side surfaced. When rough and green, they should not be more than $\frac{1}{4}''$ scant in width or thickness. Joists may be had in the following nominal sizes: $2''$, $2\frac{1}{2}''$ and $3''\times10''$, $12''$ and $14''$.

131. Measurement of Lumber. The unit of measurement of lumber is the board foot. The **board foot** is the

measure of a board an inch thick and a foot square.   Boards less than one inch thick are counted the same as though they were one inch thick.   If the boards are more than an inch in thickness they are counted at their nominal size. Finish and dimension lumber is counted at the size necessary to manufacture the piece as previously explained.

For boards one inch or less in thickness and of various widths the following table gives a convenient method of figuring board feet.

<div align="center">

TABLE XIV

BOARD FEET IN LUMBER OF DIFFERENT WIDTHS

</div>

Boards 3″ wide will contain $\frac{1}{4}$ as many board feet as they are ft. long.

      4″ $\frac{1}{3}$

      6″ $\frac{1}{2}$

      9″ $\frac{3}{4}$

      12″ wide contain as many board ft. as they are ft. long.

      15″ wide will contain $1\frac{1}{4}$ as many board ft. as they are ft. long.

      16″ $1\frac{1}{3}$

In general the following rule may be used for finding the board measure of any board, plank or timber.   *Multiply the width in feet by the length in feet by the thickness in inches.   If the width is given in inches, divide by 12 to get it in feet.*

Example.   How many board feet are there in a scantling 2″ × 4″ × 16 ft.?

$$\frac{2\times\overset{\ }{4}\times 16}{\underset{3}{12}}=\frac{32}{3}=10\tfrac{2}{3} \text{ ft. B.M.} \qquad Ans.$$

Explanation.   To change the width to feet we must divide the 4″ by 12 but this may be done by expressing the operation and canceling.   Thus 4 divided by 12 is $\frac{1}{3}$ and the result is $\frac{32}{3}$ or $10\frac{2}{3}$ ft. B.M.

132. Counting Lumber.   In the yard, at the car or on the dock, the tallyman is expected to count the lumber.   His record is kept in a small book called the *tally book*.   The

Fig. 48.—Kitchen Cabinet.

tally is recorded in different ways, depending upon the kind of material.   Framing lumber and other lumber of specified lengths is generally tallied by the piece.   Shiplap, sheathing,

ceiling and flooring is generally tied in bundles containing a certain number of feet B.M. and tallied by bundles.

133. Flooring. In figuring quantities of lumber required to cover surfaces, we must allow for matching and waste. A $1'' \times 4''$ flooring board will show only $3\frac{1}{4}''$ face. We must, therefore, allow sufficient quantity to make up for this difference. It is necessary to add about 20% in board measure to the surface measure for this purpose and from 5% to 10% for waste.

Example. How many board feet of $1'' \times 4''$ flooring will be required to lay a floor 28 ft. wide by 62 ft. long, allowing 25% for matching and waste?

$$
\begin{array}{r}
62' \\
28' \\
\hline
496 \\
124 \\
\hline
1736 \text{ sq.ft.} \\
.25 \ \% \\
\hline
86\ 80 \\
347\ 2 \\
\hline
434.00 \quad \text{Add.} \\
1736 \\
\hline
2170 \qquad \text{ft. B.M.} \qquad \qquad Ans.
\end{array}
$$

Explanation. The number of square feet floor surface is found by multiplying the length 62 ft. by the width 28 ft. to get 1736 sq.ft. 25% of this is 434 sq.ft. which when added to the actual area gives 2170 ft. B.M. of flooring required.

134. Shingles. Shingles are figured by the thousand. A *thousand* common shingles are equal to 1000 shingles 4″ wide. They are put up in bundles of different quantities according to the thickness of the butts. Shingles 18″ long having 5 butts to 2½″ are packed five bundles to the thousand. Shingles 16″ and 18″ having 5 butts to 2″ are packed four bundles to the thousand.

Fig. 49.—Work Bench.

The common unit for measuring roof surface is the **square.** A square is a surface containing 100 sq.ft., or it is equivalent to a surface 10 feet square.

Shingles are laid so that from 4″ to 5″ of the butts are exposed to the weather. The table given below indicates the number of shingles required to cover a square of roof surface when laid with various exposures allowing some waste.

TABLE XV

AREA COVERED BY SHINGLES*

Laid to the weather.	1000 will cover square feet.	No. shingles to a square.
4″	100	1000
4¼″	110	910
4½″	120	833
5″	133	752

* Kidder's Pocket Book.

Example: How many shingles laid $4\frac{1}{2}''$ to the weather will be required to cover a roof 12 ft. by 24 ft.? How many bundles if the shingles are 5 butts to $2''$?

$$
\begin{array}{r}
24' \\
12' \\
\hline
48 \\
24 \\
\hline
288 \text{ sq.ft.} = 2.88 \text{ squares.}
\end{array}
$$

$$
\begin{array}{r}
2.88 \\
833 \\
\hline
8\ 64 \\
86\ 4 \\
2304 \\
\hline
2399.04 = 2.4 \text{ thousand shingles.}
\end{array}
$$

$$
\begin{array}{r}
2.4 \\
4 \\
\hline
9.6 \text{ or 10 bundles of shingles.}\quad Ans.
\end{array}
$$

Explanation. From the above table we find that when shingles are laid $4\frac{1}{2}''$ to the weather 833 are required per square. We must first find the number of squares of roof surface to be covered. There are 288 sq.ft. or 2.88 squares. At 833 shingles per square, it would require about 2400 shingles. Counting 4 bundles to the thousand, it would require 10 bundles to cover the roof.

Fig. 50.—Roof.

135. Material Lists. The builder is frequently called upon to make out material lists or lumber bills for various structures. In doing this work it is imperative that it should be done in an orderly manner. The different kinds of material should be grouped and their use in the structure designated. A reference mark on the drawing and a corresponding mark on the computations will often serve to connect the two. The results should be recorded in such a way that they may be readily understood at any time.

If the habit of neat and orderly work in making out lumber bills and purchasing lists is formed early it will be of great value during the entire business career of the builder. The following example will show a convenient method of making out lumber bills.

Example: Make out the lumber bill for the private garage shown in Fig. 43.

A = 1 pc 4×4×6' S. 1 S. 1 E.
      1 pc 4×4×12'   "   } Sill
      2 pcs 4×4×18'   "

B = 30 pcs 2×4×10' S. 1 S. 1 E.   Posts

C = 2 pcs 2×4×12' S. 1 S. 1 E.   } Plate
      2 pcs 2×4×18'   "

D = 28 pcs 2×4×10' S. 1 S. 1 E.   Rafters

E = 1 pc 2×6×22' S. 2 S.   Ridge

F = 3 pcs 2×4×14' S. 1 S. 1 E.   Belt

G = 2 pcs 2×4×10' S. 1 S. 1 E.   Headers

H = 375 ft. B.M. 1×6 S. 1 S.   Sheathing

I = 14 bundles shingles

J = 700 ft. B.M. $\frac{3}{4}''×5\frac{1}{2}''$ drop siding

K = 4 pcs 2×6×10' S. 1 S. 1 E.   } Stiles and rails
      2 pcs 2×6×12

Explanation. For most of the material in this list it is only necessary to take the quantities off of the plan, adjusting the lengths to the proper stock sizes. Some judgment must be exercised in making this adjustment, but one soon becomes accustomed to the work and can tell at a glance the proper lengths to specify. Since so much does depend upon judgment one man's list will not be exactly like another's on the same structure. The sheathing is figured as previously explained, allowing 10% for waste

and adding sufficient material for the ridge board.   Twenty per cent is allowed for lapping and waste in figuring the siding.   It is always better to allow for liberal quantities rather than to run short of material on the job.

This is not a purchasing list.   In sending the list to

Fig. 51.—Garage.

the lumber dealer the items should be arranged so that all the material of a given kind and length is grouped together.   This material list should be carefully preserved and placed in the hands of the foreman when the material is delivered on the job so that he will know without question just where every stick belongs in the structure.

Fig. 52.—Shed.

SUMMARY OF CHAPTER XII

76. To find the number of feet board measure in a piece of lumber, multiply the width in feet by the length in feet by the thickness in inches.   (Sec. 131.)

77. To find the number of feet board measure of flooring to cover a given floor, find the area of the floor in square feet and add 25% to 30% for matching and waste.   (Sec. 133.)

78. To find the number of shingles required to cover a given roof surface, find the size of the surface in squares and multiply by the number of shingles required per square as given in the table.   Multiply the number of thousand of shingles thus obtained by 4 or 5 according to the number of bundles to the thousand.   (Sec. 134.)

PROBLEMS

182. How many board feet in a piece of lumber $1'' \times 8''$, 14' long?

183. Six pieces of finish lumber $1\frac{1}{2}'' \times 8''$, 12' long are required for a certain job.   How many board feet are there in the lot?

184. A contractor orders the following bill of dimension stuff for concrete forms.   How many feet B.M. are there in each lot?

> 24 pcs. 4×4 S. 1 S. 1 E. 16' long
> 180 pcs. 2×4 S. 1 S. 1 E. 10' long
> 200 pcs. $1\frac{1}{4} \times 8$ S. 1 S. 1 E. 12' long.

185. How many board feet are there in 200 pieces $2'' \times 14''$ joists 16' long?

186. How many board feet of sheathing $1'' \times 6''$ are required to cover the side of a barn 12' high and 20' long, allowing 10% for waste?

187. How many feet B.M. of sheathing $1'' \times 6''$ are required to sheath a roof surface 16' wide and 32' long, allowing a 2'' space between boards and 10% for waste?

FIG. 53.—Barn.

END VIEW

SECTION

2″ x 6″ rafters
2′ 0. o.c.

2″ x 6″ plate
1″ x 6″
D. & M. flooring

2″ x 6″ Studs
20″ o.c.

2″ x 6″ Sill

2″ x 10″ Joi
16 o.c.

10′

ELEVATION

Shingles
5″ to the weather

Ship lap

1 x 6 belt

Ship lap
1″ x 8″

9′-6″

6″

10″

7′

PLAN

16′

10 x 12 - 8 Lt.

Grain
Bin
above

flooring 5′ high
1″ t. & g.

Cement Floor

Ship lap

Harness

Horse
Stalls

2 x 8 Plank
Floor

2 x 8 Partition

10′

26′

7′

Cement   Floor

1½′

188. A man wishes to build a sidewalk 6′ wide and 150′ long. The planks are $1\frac{1}{2}''\times 8''$ S. 1 S. 1 E. He puts in 3 sleepers $4''\times 4''$ under the planks for nailing. How many board feet must he order of each lot, allowing 10% waste?

189. How many board feet of $1''\times 3''$ tongue-and-groove flooring will be required to lay a floor $14'\times 16'$, allowing for face measure and 5% for waste?

190. Fig. 44 shows the first-floor plan of a small cottage. How many feet B.M. of $1''\times 4''$ flooring will be required, including the porches, allowing 5% for waste?

191. Fig. 45 shows the plan of an ordinary ridge roof. How many feet B.M. will be required to sheath the roof using $1''\times 6''$ sheathing spaced $2''$ apart?

192. Fig. 46 shows the forms for a concrete job in which $1\frac{1}{4}''\times 8''$ shiplap is used. How many feet B.M. of shiplap will be required?

193. How much material will be required to make the cupboard shown in Fig. 47?

194. Figure the lumber required to make the kitchen cabinet shown in Fig. 48. Give the number of pieces and the B.M.

195. How much material will be required to make the workbench shown in Fig. 49. Give the number of pieces and the B.M.

196. How many thousand shingles will be required for the roof shown in Fig. 50, if they are laid $4\frac{1}{4}''$ to the weather?

197. How many bundles of shingles will be required for the garage shown in Fig. 51, if they are laid $4\frac{1}{2}''$ to the weather?

198. Make out the complete lumber bill in pieces for the shed shown in Fig. 52. Give the number of pieces of each different length.

199. Make out the complete lumber bill for the barn shown in Fig. 53.

CHAPTER XIII

BUILDERS' GEOMETRY.   GEOMETRIC CONSTRUC-
TIONS.   ANGLES AND ANGULAR MEASURE.
ELLIPSE.   POLYGONS.

136. Use of Geometry. The carpenter and wood-worker has to deal largely with points, straight and curved lines, plane and curved surfaces and various kinds of solids. **Geometry** is a study of the properties, construction and measurement of lines, surfaces and solids. A thorough knowledge of geometry is very useful.

137. Geometric Points and Lines. A point has no dimensions; it merely has position. In marking points with a pencil or scriber, we must give them some size in order to see them, but theoretically they are only imaginary.

A line has only one dimension, that of length. A line drawn with a pencil or chalk has some width, but in theory it should have no width. Lines may be straight or curved. **A straight line** is one that does not change its direction. **A curved line** is one which changes its direction at every point. **A broken line** consists of a series of straight lines variously directed and joined together.

138. Geometric Surfaces and Solids. A surface has two dimensions—length and breadth. **A plane surface** is one which will wholly contain a straight line no matter in which direction the line is laid in the plane. **A curved surface** is one which changes its direction in accordance

with a given law.   The surface of a cylinder or sphere is a curved surface.

The definition of a plane surface gives a practical method of testing for a true plane.   If a surface has a warp or wind, it may easily be detected by laying a straight-edge on the surface in different positions and sighting for unevenness.   If the plane is true, the straightedge will lie wholly in the surface in whatever position it may be placed.   This method of testing is illustrated in Fig. 54.

Fig. 54.—Testing a Surface.

A solid is *space* completely surrounded by surfaces. A solid has length, breadth, and thickness.

139. Geometric Angles.   An angle is formed by two straight lines which meet at a point.   The point is called the *vertex* of the angle.   **A right angle** is one in which the two intersecting lines are perpendicular to each other. The angle between the two edges of a steel square is a right angle.   **An acute angle** is one in which the lines make less than a right angle.   **An obtuse angle** is one in which the lines make more than a right angle.

An angle is designated by letters placed at the point or vertex and on the sides or legs.   The letter at the vertex is always written *between* the letters representing the sides.

In Fig. 55 the angle *DAB* is a right angle, as is also the angle *BAC*, for the reason that the line *AB* is *perpendicular to* the line *DC* at the point *A*. The angles *EOF* and *GOH* are both acute angles because each is *less than* a right angle. The angles *EOG* and *FOH* are both obtuse angles because each is *greater than* a right angle.

140. Circular or Angular Measure. In Chapter X the circle was studied in part and some of its properties were investigated. In this chapter additional definitions and properties will be considered. **The arc of a circle** is a

FIG. 55.

part of its circumference. The part of the circumference between the points *A* and *C* in Fig. 56 is called the *arc ABC*. **A chord** is a straight line connecting two points on the circumference of a circle. The straight line joining the points *A* and *C* in Fig. 56 is spoken of as the *chord ADC*.

Circular arcs are measured in **degrees**. In a complete circle there are 360 degrees. In a fourth of a circumference or **quadrant**, there are 90 degrees of arc. An angle at the center is measured by the number of degrees of arc intercepted between the sides of the angle; the number of degrees in the arc gives the number of degrees in the angle. For precise measurements degrees are divided into **minutes** and minutes are again divided into **seconds**.

There are 60 seconds in one minute and 60 minutes in one degree. Degrees, minutes, and seconds are designated by the symbols °, ′, ″, respectively. Thus 34 degrees, 42 minutes and 30 seconds are written 34° 42′ 30″.

141. Sectors and Segments. The area included between two radii and the arc is called a **sector.** In Fig. 57 the area *AOBC* is a sector of the circle. **A segment** is that part of a circle which is included between an arc

FIG. 56.—Chord and Arc.   FIG. 57.—Segment.

and its chord. Thus in Fig. 57 the shaded portion *ACBD* is a segment.

142. To Bisect a Line and to Erect a Perpendicular. To bisect a line means to divide it into two equal parts. Ordinarily it would only be necessary to measure the total length of the line, divide this measure by two and lay off the half distance on the line. By the methods of geometry we not only can bisect a line without actually measuring it, but we can also erect a perpendicular line, or a line making a right angle with the first, at the middle point.

Let AC, Fig. 58, be the line of which we wish to find the middle point. Using the ordinary carpenter's compass, set the points a little more than half the length of the line AC apart. With this distance as a radius and the points A and C as centers, strike two arcs so that they cross above and below the line AC at D and E. Through these points draw the line DE. The line DE will bisect AC at the point F and is also perpendicular to AC.

Fig. 58.—Perpendicular Bisector.

DE is called the *perpendicular bisector* of AC.

143. To Bisect an Angle. Let AOB in Fig. 59 be an angle which is to be divided into two equal angles. With

Fig. 59.—Bisecting an Angle.

O as a center and any convenient distance as a radius, draw an arc which cuts the legs of the angle at C and D.

Then, with C and D as centers, and any convenient radius strike two arcs. These will intersect at the point E. Join O and E. The line OE divides the angle into two equal parts because it is everwhere equally distant from the lines OA and OB.

144. To Erect a Perpendicular at any Point on a Line. We have already learned how to erect a perpendicular bisector to any line, but suppose we wish to erect a perpendicular to a line at a given point on the line. Let DB, Fig. 60, be the line on which we desire to erect a per-

Fig. 60.—Erecting a Perpendicular.

pendicular at the point C. From any convenient point above the line, such as O, strike an arc passing through C, the given point. It will also cut DB at another point F. From F draw a line through the center O and extend it to cut the arc at A. Join A and C. This line AC will be perpendicular to DB.

145. To Construct a Right Angle. The following method, known as the " 3, 4, 5 " method, is frequently used to lay out a right angle.

On the line AC in Fig. 61 measure three units, say three feet, from A to E. With A as a center strike an arc four units long. With E as a center strike an arc five units

long. They will intersect at D. Join D and A. AD will make a right angle with AC. In other words the line AB, which is a continuation of AD, will be *square* with AC.

The reason for the above is that a right triangle is formed. We have learned that in a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides. If this is true, then $3^2 + 4^2$ must be equal to 5^2. $3^2 + 4^2 = 25$. Also, 5^2 equals 25, which estab-

FIG. 61.—Constructing a Right Angle.

lishes the truth of the proposition. Notice that any multiple of the units 3, 4, and 5 may be used, such as 9, 12, and 15, or 12, 16, and 20.

In laying out foundations, **batter boards** are set up and lines established as shown in Fig. 62. The accuracy of the right angle at the corner may be tested by using the " 3, 4, 5 " method. A cloth measuring-tape is held with the zero mark at A, the intersection of the lines. Measure out 9 ft. to the point E and let out 27 ft. of tape. Grasp the tape at a point 15 ft. from E and secure the 36 ft. mark at A. Draw the tape taut and a right angle

will be formed the sides of which are 9', 12', and 15' long. The lines may then be established square with each other.

Fig. 62.—Laying out Foundations.

146. To Construct Various Angles. A right angle may be constructed as directed above or it may be laid out by means of the steel square. An angle of 45° may be obtained by bisecting a right angle. It may also be constructed by drawing a diagonal of a square. To lay out a 60° angle, draw a horizontal line, as AB in Fig. 63. With AB as a radius and A and B as centers, scribe arcs intersecting at C. Draw the line AC and the angle BAC will be 60°. If BC is drawn, each of the angles will be 60°. A 30° angle may be found by bisecting a 60° angle.

Fig. 63.—A 60° Angle.

147. To Find the Center of an Arc or Circle. The carpenter often finds practical cases in which it is necessary

to find the radius of a circle when the segment or arc is given. In Fig. 64 it is desired to find the radius of the circle of which the arc is a part. From any point on the arc, such as A, draw the two chords interesecting the arc in any two points as B and C. With the points A, B, and C as centers and any convenient radius, draw the short intersecting arcs dd', ee', ff', and gg'. Through the points

Fig. 64.—Finding the Radius of a Circle

in which these arcs intersect, draw the lines MO and NO. These lines will be the perpendicular bisectors of the chords AC and AB and they will intersect each other at the point O, which is the center of the required circle. The distance from O to the given arc is the required radius.

148. The Ellipse. The practical man sometimes has occasion to use the ellipse in laying out work. Such work is most usual in the construction of concrete forms for

sewers and arches.   There are several methods of construct-
ing an ellipse, but the most convenient is by means of the
trammel.

To construct an ellipse by the trammel method, having
given the length and width of the ellipse, lay out the long
and short axes at right angles to each other as shown in
Fig. 65.   The short axis
CD is called the *minor axis*
and the long axis AB is
the *major axis.*   With a
straightedge or the tram-
mel, using three trammel
points, lay off the distance
from O to D, or one-half
the minor axis between
the points a and b on the
straightedge. From a, again

Fig. 65.—Constructing an Ellipse.

lay off one-half the major axis or the distance from O to
B.   This will locate the point c on the straightedge.   Now
keep the point b on the major axis and the point c on the
minor axis and locate several positions of the point a by
making a mark in the wood or paper as shown in the figure.
Join the points thus found with a smooth curve and the
result will be an ellipse.   The points should be located
fairly close together in order to make the curve a smooth one.

149. **Polygons.**   A polygon is a plane figure bounded
by any number of straight lines.   The **side** of a polygon
is any one of these lines.   The point where two lines meet
is called a **vertex.**   A polygon is designated by reading the
letters at the vertices.   **A triangle** is a polygon which has
three sides.   **A quadrilateral** is a polygon which has four
sides.   **A parallelogram** is a quadrilateral whose opposite
sides are parallel to each other.   Fig. 66 shows a parallel-

ogram. When all the angles are right angles and the
sides are all equal the figure is a **square**. When all of the

Fig. 66.—A Parallelogram.

angles are right angles but
when the opposite sides
only are equal to each
other, the figure is a **rect-
angle**. **A hexagon** is a
polygon having six equal
sides and six equal angles.
An octagon is a figure
having eight equal sides and eight equal angles.

150. To Construct a Hexagon. To construct a hexagon
in a given square, proceed as follows: First, find the center
of the square by drawing a diagonal from corner to corner
as shown in Fig. 67. Then inscribe a circle within the square

Fig. 67.
Constructing a Hexagon.

Fig. 68.
Constructing an Octagon.

so that the circumference of the circle just touches the sides
of the square. With the center of one side of the square,
the point *A* for example, as a starting point, and with the
length of the radius of the circle as a measure, step off
around the circle. This should divide the circle exactly

into six equal parts.   Now join these points just found with a straight line to form the hexagon *ABCDEF*.

151. To Construct an Octagon.   The octagon or eight-sided figure may be constructed as shown in Fig. 68.   Find the center of the square as explained above.   Then with the distance from *O* to *A* as a radius and with the points *A*, *B*, *C*, and *D* used one after the other as centers, draw arcs to cut the edge of the square in the points *a*, *a'*, *b*, *b'*, etc.   Join these points to form the octagon.

SUMMARY OF CHAPTER XIII

79. Geometry is a study of the properties of lines, surfaces, and solids.   (Sec. 136).

80. A point is a geometric representation which has position only.   (Sec. 137).

81. A line is a geometric representation which has direction and length.   Lines may be straight or curved. (Sec. 137).

82. A surface is a geometric representation which has length and breadth, but no thickness.   Surfaces may be either plane or curved.   (Sec. 138).

83. A solid is a geometric representation which has length, breadth, and thickness.   (Sec. 138).

PROBLEMS

Note.   To lay out the construction asked for in the following problems, use a small steel square or wooden or celluloid triangle and a compass or dividers.   A cheap compass which may be attached to a lead pencil may be obtained at any book store.   Use a fairly hard lead pencil so that your work will not smear and get dirty.

200. Construct a square exactly 4″ on a side and square the corners by the method given in Section 144.

201. Two lines are stretched for the corner of a concrete foundation.   Show how you can test them by the "3, 4, 5," method. Make a diagram.

202. Construct angles of 60°, 45°, and 30°.

203. Construct the largest hexagon you can draw in a 4″ square.

204. Construct the largest octagon you can draw in a 4″ square.

205. Construct a hexagon each side of which is $2\frac{1}{2}$″ long.

Hint.   The radius of the circle in which it is inscribed must be $2\frac{1}{2}$″.

206. Draw an ellipse by the trammel method, using a straight piece of paper as a straightedge.   Make the ellipse 3″ wide and 5″ long.

207. Lay out a circle 4′ in diameter in six segments so that it may be built from 8″ boards. Make your drawing to a scale of $1\frac{1}{2}″ = 1′$. How long must the pieces be cut to make each segment?

208. Draw a circle 4″ in diameter and lay out in it a sector of which the angle at the center is 60°.

209. A window is to be constructed in the form of a half circle or semicircle with a radius of $1\frac{1}{2}$ ft. The glass is to set in lead. At the center is a small semicircle with a 3″ radius and the rest of the glass area consists of six sectors. Make a drawing showing the size and shape of all the pieces in the window.

AREAS OF SURFACES AND VOLUMES OF SOLIDS. AREA OF TRIANGLE, PYRAMID, CYLINDER AND CONE. VOLUME OF CYLINDER, PRISM, PYRAMID AND CONE. MEASURING SURFACES AND VOLUMES

WE have already learned how to find the area of the square and rectangle and also how to find the length of the hypotenuse of a right-angled triangle. The student should review these subjects carefully before taking up this chapter.

FIG. 69.

152. Area of a Triangle. Fig. 69 shows how two triangles are formed by drawing a diagonal line from one corner to the other of a rectangle. You can readily see that the area of each of these triangles will be one-half of the area of the rectangle. If the letter S stands for the area of the rectangle, L for its length, and W for its width; then,

$$S = L \times W.$$

The area of each triangle is one-half the area of the rectangle. Let the letter s stand for the area of the triangle, b for its base and a for its altitude; then

$$s = \tfrac{1}{2}S = \tfrac{1}{2}b \times a.$$

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If you look at the rectangle you will now see that the length of the rectangle is the same as the altitude of the triangle and that the width of the rectangle is equal to the base of the triangle. We may now formulate the rule that the *area of a triangle is equal to one-half the product of the base by the altitude.* Please note that, as in the case of finding the area of the square and of the rectangle, the altitude and the base *must be expressed in the same units.*

Example. Find the area of one triangle in Fig. 69 if $L = 30''$ and $W = 20''$.

$$\begin{array}{r} 20'' \\ 30'' \\ \hline \end{array}$$

2)600 sq.in.

   300 sq.in. area of triangle.   *Ans.*

Explanation. The area of a rectangle 30″ in length and 20″ in width equals $20'' \times 30'' = 600$ sq.in. The area of the triangle formed on these sides is one-half the area of the corresponding rectangle. Hence, we divide this product by 2 and get 300 sq.in as the area of the triangle.

153. Area of a Pyramid. A solid whose base is a polygon and whose sides are all triangles which meet in a common point above the base is called a **pyramid.** The common point in which the sides meet is called the **vertex** or the **apex.** When the vertex is perpendicularly above the center of the base the pyramid is a **right pyramid.** If the upper portion of a pyramid is removed, as by a saw cut parallel to the base, the portion that remains is called the **frustum** of the pyramid. Pyramids are named according to the number of sides there are in the base. If the base is a square, it is called a square pyramid. If the base is a hexagon, it is called a hexagonal pyramid. Various forms

of the pyramid are often found in roofs, walls, bins, hoppers, etc. Fig. 70 is an example of the hexagonal pyramid.

Since the sides of a pyramid are triangles, we can use the information given above to find the surface area of a

FIG. 70.—Hexagonal Pyramid.   FIG. 71.—Square Pyramidal Roof.

pyramid. An example applying to a pyramidal roof is given below.

Example. Find the number of feet B.M. necessary to cover the square pyramidal roof shown in Fig. 71.

$$7^2 = 49$$
$$7^2 = 49$$
$$\overline{98}$$

$$\sqrt{98} = 9.9' = \text{length of } a.$$

9.9'
  · 7'
_____

69.3 sq.ft. in one side
   4 sides
_____

277.2 sq.ft. in roof
   20% for waste
_____

55.440

277.2

55.4
_____

332.6 sq.ft. or say 335 ft. B.M. of lumber required.

Ans.

Explanation. Since the surface we are measuring is inclined, the altitude of the triangle the area of which we require is the line a. This length is found by solving for the hypotenuse of a triangle when the base and the altitude are given. The altitude in this case is the line $EO = 7'$ and the base is $AE = \frac{14}{2} = 7'$. Solving the triangle we get 9.9' as the length of the line a. When we lay the side of the roof out flat, it appears as shown in Fig. 72. The altitude is 9.9' and the base is 14'. If we divide 14 by 2 before multiplying it will amount to the same thing as dividing the product by 2. Multiplying 9.9' by 7' gives 69.3 sq.ft. for the area of the triangle. Since there are four sides, we

Fig. 72.

multiply by 4 to get the total area of the roof. Allowing 20% for waste gives 332.6 sq.ft or say 335 ft. B.M. of lumber required.

154. Area of a Cylindrical Surface. A **cylinder** is a solid whose ends or bases are two parallel circles and the side of which is a curved surface. A cylinder may be formed by revolving a rectangle about one of its sides as an axis, in which event the cylinder is a **right cylinder**. Fig. 73 shows a right cylinder. We have many familiar examples of the use of cylinders in practical building construction.

If we should roll a cylinder out on a flat surface as shown in Fig. 74, it would form a rectangle the width of which would equal the height of the cylinder and the length of which would be equal to the circumference of the base of the cylinder. Therefore, *to find the area of the surface of a cylinder, find the circumference of the base and multiply by the height of the cylinder.* To find the total area add to this the area of the two bases.

Fig. 73.—Cylinder.

We have already learned that the circumference of a circle is equal to π times the diameter and that the area of a circle is equal to the radius squared times π. Let h equal the height of the cylinder and r equal the radius. Then the diameter d equals $2r$. If s represents the area of the surface; then

$$s = 2\pi r h = \pi d h.$$

If S represents the total area, then,

$$S = 2\pi rh + 2\pi r^2 = \pi dh + \tfrac{1}{2}\pi d^2.$$

Example. Find the number of feet B.M. required to build the walls of a wooden stave silo 12′ in diameter and 30′ high. Plank 2″ thick are to be used. Add 20% for matching and cutting.

$$3.1416$$
$$12' \; d = 2r$$

$$6\;2832$$
$$31\;416$$

37.6992 ft. circumference.

37.7′
   30′ high

1131.0 sq.ft. in surface.

1131
   2 for 2″ plank.

2262

2262
   20% for matching and cutting.

452.40

  2262
   452

2714 ft. B.M. of lumber required.   *Ans.*

Explanation. Multiply the diameter by π to get the circumference. This times the height in feet will give the number of square feet in the surface. If 2″ plank are used

and 20% allowed for matching and cutting, the total number of feet B.M. required will be about 2700.

155. Area of the Surface of a Cone. A **cone** is a solid the base of which is a circle and the side of which is curved surface which closes in to a point called the **vertex**. The distance from the vertex to a point on the circumference of the base is called the **slant height**. A cone may be formed by revolving a right triangle about its altitude as an axis.

FIG. 74.

When the upper portion of a cone is cut away as with a saw cut, the part that remains is called a **frustum** of a cone. Fig. 75 shows a cone and a frustum. This surface is found in practical construction in the roofs of round buildings and towers.

If we should roll a cone out upon a flat surface as we did for the cylinder, we would get a part or sector of a circle. This is shown in Fig. 76. The length of the arc AB, Fig. 76, is equal to the circumference of the base of the cone. The radius of the sector is equal to the length of the slant height or the line a on the cone. This length may be found by

solving for the hypotenuse of the triangle OEK, Fig. **75**. The area of this sector is equivalent to the area of a triangle which has a base equal to the length of the arc AB and an altitude equal to the slant height of the cone. If we let s equal the lateral area of the cone, r the radius of the base and a the slant height, then

$$s = \tfrac{1}{2} \cdot 2 \cdot \pi r a = \pi r a.$$

Fig. 75.—Cone and Frustum.

Just as in the triangle it is equal to one-half the circumference of the base times the slant height. The circumference of the base is equal to $2\pi r$ and canceling the 2 with $\tfrac{1}{2}$ the formula reads $\pi r a$. In other words this is: *The lateral area of a cone is equal to the product of π by the radius of the base and by the slant height.* If the total area S is wanted, it is equal to $S = \pi r a + \pi r^2$, and is found by adding the area of the base circle to the lateral area.

Example. How many shingles laid $4\frac{1}{2}''$ will be required to cover the roof of the silo shown in Fig. 77?

$$7^2 + 7^2 = 98$$
$$\sqrt{98} = 9.9' \text{ slant height the roof.}$$

$$
\begin{array}{r}
3.1416 \\
7' \\
\hline
21.9912 = \quad 22' \\
9.9 \\
\hline
198 \\
198 \\
\hline
217.8 \text{ sq.ft. in roof.}
\end{array}
$$

$$
\begin{array}{l}
217.8 \\
10\% \text{ for waste} \\
\hline
21 \quad \text{add for waste} \\
217.8 \\
\hline
238.8 \text{ sq.ft. or say } 2\frac{1}{2} \text{ squares.} \\
833 \\
2\frac{1}{2} \\
\hline
416 \\
1666 \\
\hline
2082 \text{ shingles required.} \\
4 \text{ bundles to the thousand}
\end{array}
$$

8.328, say 9 bundles of shingles required.   *Ans.*

Explanation. Solve the triangle to find the slant height of the roof. Find the product of π times the radius of the base and times the slant height. Add 10% for waste and count 4 bundles of shingles to the thousand.

156. Volume. We have already learned something about cubic measure and how to find the volume of a cube. It is sometimes necessary to know how to find the cubic contents of bins and other structures, having various shapes. It will usually be possible to separate an irregularly formed structure into parts which are, or very nearly are, classed under one of the regular forms. We should, therefore, know how to figure the volume of the cylinder, the pyramid and its frustum and the cone and its frustum.

FIG. 76.

157. Volume of a Cylinder. We have already learned that the volume of a cylinder is equal to the product of the area of the base by the height of the cylinder. We can now express this as a formula. Let V equal the volume of the cylinder, r the radius of the base and h the height of the cylinder; then

$$V = \pi r^2 h.$$

Example. How many cubic feet in a concrete silo having 12 ft. inside diameter and 30 ft. high?

$$6^2 = 36$$

$$36 \times 3.1416 = 113 \text{ sq.ft. in bottom.}$$

$$113 \times 30 = 3390 \text{ cu.ft. in silo.}$$ *Ans.*

Fig. 77.—Silo.

Explanation. Find the area of the bottom of the silo by squaring the radius and multiplying by π. Multiply this by the height to get the cubic contents.

158. Volume of a Prism. A solid whose ends are parallel polygons and whose sides are parallelograms is called a

prism. When the edges of the sides make right angles with the edges of the base, the prism is a **right prism.** Our most familiar example of a right prism is simply a square or rectangular block of wood. *The volume of a prism is equal to the product of the area of the base by the height.*

Fig. 78.—Three Pyramids Cut from a Cube.

159. Volume of a Pyramid. Fig. 78 shows how three pyramids may be cut out of a cube. This demonstrates that a pyramid contains one-third as much as the corresponding cube or prism. Therefore, *the volume of a pyramid is equal to one-third the product of the area of the base by the altitude.*

If a represents the length of one side of the base, b the length of the other side, h the altitude, then if V represents the volume,

$$V = \tfrac{1}{3}abh$$

FIG. 79.—Bin with Hopper.   FIG. 80.—Gas Container.

Example. Fig. 79 shows a bin with an inverted pyramid for a hopper at the bottom. Find the total contents of the bin in bushels.

For the volume of the upper prism:

$$10' \times 10' \times 20' = 2000 \text{ cu.ft.}$$

For the volume of the hopper:

$$\tfrac{1}{3} \times 10' \times 10' \times 8' = 266 \text{ cu.ft.}$$

For the total volume:

$$2000 + 266 = 2266 \text{ cu.ft.}$$

There are 1.25 cu.ft. in a bushel,

$$2266 \div 1.25 = 1813 \text{ bu.} \qquad \qquad Ans.$$

Explanation. To find the contents of the upper bin, multiply the length by the width and again by the height. To find the contents of the hopper take one-third of the product of the area of the base by the altitude. Adding this to the quantity for the upper portion we have 2266 cu.ft. for the total contents of the bin. There are $1\frac{1}{4}$ cu.ft. in each bushel; hence we divide the contents in cubic feet by $1\frac{1}{4}$ to get 1813 bushels.

160. Volume of a Cone. The method of finding the volume of a cone is the same as that for finding the volume of a pyramid. *It is equal to one-third the product of the area of the base by the height.* The volume is

$$V = \tfrac{1}{3}\pi r^2 h,$$

in which r is the radius of the base and h is the height or altitude.

Example. A gas container is in the form of a cylinder topped by a cone. The dimensions of the container are given in Fig. 80. How many cubic feet of gas will the container hold?

Note. Squaring the diameter and multiplying by .7854 is the same as squaring the radius and multiplying by 3.1416.

$$5^2 = 25$$

$$25 \times .7854 = 19.63 \text{ sq.ft.}$$

$$19.63 \times 8 = 157 \text{ cu.ft. in the cylinder.}$$

$$19.63 \times 2.5 = 49.07.$$

$$\tfrac{1}{3} \times 49.07 = 16.35 \text{ cu.ft. in cone.}$$

$$157 + 16.35 = 173.35 \text{ cu.ft. total.} \qquad Ans.$$

Explanation. Find the contents of the cylinder in the usual way. Find the contents of the cone by taking $\tfrac{1}{3}$ of

Fig. 81.—A Deck Roof

the product of the area of the base, 19.63 cu.ft., by the altitude, $2\tfrac{1}{2}$ ft., to get 16.35 cu.ft. Add the contents of the cylinder to the contents of the cone to get 173.35 cu.ft. as the total contents of the container.

161. Volume of the Frustum of a Cone or Pyramid. A frustum of a cone or pyramid is figured by first finding what the contents would be if a part were not cut away.

Then find the volume of the part cut away and subtract from the whole volume to get the volume of the frustum.

162. Measuring Surfaces and Volumes. Many of the cases which one finds in practical work do not fit these regu-

FIG. 82.—Building with Gambrel Roof.

lar forms precisely. It is necessary, then, to divide the whole into a number of parts which will come into one of these classes. The accuracy of the result will depend largely upon the judgment used in making these divisions and skill in carrying out the operation.

Example. Fig. 81 shows a deck roof and gives the dimensions. Figure the total area of the roof.

Slant height of roof

$$8^2 = 64$$
$$8^2 = 64$$
$$\overline{}$$
$$128 = 11.31'$$

Area of 8 small triangles A, A, A, etc.:

$$\frac{11.31 \times 8}{2} = 45.24 \text{ sq.ft. in one triangle.}$$

$45.24 \times 8 = 361.92$ sq.ft. in 8 triangles.

Area of 2 rectangles B, B:

$11.31 \times 18 \times 2 = 407.16$ sq.ft.

Area of 2 rectangles C, C:

$11.31 \times 4 \times 2 = 90.48$ sq. ft.

Area of rectangle D:

$18 \times 4 = 72$ sq.ft.

Adding:
$$
\begin{array}{r}
361.92 \\
407.16 \\
90.48 \\
72. \\
\hline
931.56 \text{ sq.ft. total.} \qquad Ans.
\end{array}
$$

Explanation. Divide the roof surface up into triangles and rectangles. Figure the area of each of these separately and then add them all together to get the total area.

SUMMARY · OF CHAPTER XIV

84. **The area of a triangle** is equal to one-half the product of the base by the altitude. $S = \frac{1}{2} ba.$ (Sec. 152.)

85. **The lateral area of a cylinder** equals two times π times the radius of the base times the height. $S = 2\pi rh.$ (Sec. 154.)

86. **To get the total area of a cylinder** add to the lateral area the area of both bases. $S = 2\pi rh + 2\pi r^2.$ (Sec. 154.)

FIG. 83.

87. **The lateral area of a cone** equals π times the radius of the base times the slant height. $S = \pi ra.$ (Sec. 155.)

88. **To get the total area of a cone** add to the lateral area the area of the base. $S = \pi ra + \pi r^2.$ (Sec. 155.)

89. **The volume of a cylinder** is equal to π times the square of the radius of the base times the height. $V = \pi r^2 h.$ (Sec. 157.)

90. **The volume of a prism** is equal to the product of the area of the base by the height.   (Sec. 158.)

FIG. 84.—Framing for a Hip Roof.

91. The volume of a pyramid is equal to one-third the product of the area of the base by the height. $V = \frac{1}{3}abh$. (Sec. 159.)

92. The volume of a cone is equal to one-third the area of the base by the height. $V = \frac{1}{3}\pi r^2 h$. (Sec. 160.)

Fig. 85.

PROBLEMS

210. A triangle has a base of 20′ and an altitude of 15′. What will be the side of a square having the same area?

211. Fig. 82 shows the end view of a building with a gambrel roof. Figure the area of the walls and area of the roof.

212. In the problem above how many feet B.M. of $1'' \times 8''$ shiplap will be required to sheath the walls? How many feet B.M. of $1'' \times 4''$ laid $2''$ apart will be required to sheath the roof? How many shingles will be required for the roof?

213. In Fig. 83 find the length of rafters, struts and other parts not given.

214. Fig. 84 shows the framing for a hip roof. How many feet B.M. will be required if $2'' \times 4''$ are used for jack rafters and $2'' \times 6''$ for hip and ridge?

FIG. 86.—Roof for a Pavilion.

215. An octagonal tower is $20'$ across from side to side. The height of the apex is 11 ft. above the eaves. How many shingles will be required for the roof? Draw the octagon to scale to find the length of a side.

216. How many cubic feet in a monolithic concrete silo $16'$ in diameter on the outside and $45'$ high, if the walls are $9''$ thick?

217. Find the number of shingles required for the roof shown in Fig. 85.

218. How many square feet of surface in a conical roof $15'$ in diameter at the base and $8'$ high?

219. Fig. 86 shows a roof for a pavilion. If the shingles cost $5.00 per thousand and they are laid $5''$ to the weather what will be the cost of shingling the roof? Figure $3.00 per M shingles for labor and nails.

CHAPTER XV

USE OF THE CARPENTER'S SQUARE. BRACE MEASURE. OCTAGONAL SCALE. ESSEX BOARD MEASURE. DIAGONAL SCALE. POLYGON SCALE. GEOMETRIC APPLICATIONS

163. Carpenter's Square. The steel square or carpenter's square is probably the most useful tool which the carpenter has in his kit. In the usual form it consists of blade or " body " 24 in. long and 2 in. wide and a " tongue " 16 in. long and $1\frac{1}{2}$ in. wide. The tongue makes an exact right angle or an angle of 90° with the blade. Both the blade and the tongue are graduated in inches and fractions of an inch. These graduations begin at the corner and run out on both edges of the tongue and blade. The fractional graduations are usually in thirty-seconds, sixteenths, twelfths, tenths and eighths. The square is also supplied with a brace measure, an octagonal scale, the Essex board measure and a diagonal scale, the uses of each of which will be explained. Some squares give a rafter table instead of the Essex board measure scale. The side of the square which has the maker's name stamped upon it is called the **face** and the opposite side is called the **back**. Fig. 87 shows the face of a steel square and Fig. 88 shows the back.

164. The Brace Measure. The brace measure is used to determine the length of a diagonal brace. When the length of two sides of a square are known, the length of the diagonal may be found by using the brace measure. The

figures along the center of the back of the tongue as shown in Fig. 88 constitute the brace measure. The e q u a l figures, one above another, repre-

sent the side of the square and the figure to the right and between them represents the length of the diagonal.

Example. What is the length of a diagonal brace for a square 48″ on a side?

Explanation. Look on the brace measure to find $\frac{48}{48}$. Between these two figures and the right you will find 67.88. This the length of the required brace in inches and decimals.

165. The Octagonal Scale. The octagonal scale is sometimes called the " eight square scale." It is shown along the center of the face of the tongue in Fig. 87. To use the scale you take as many divisions from the scale as there are inches in the width of the stick. Lay this distance off on each side of the center line of the end of the stick after it has been carefully centered. Connect these points diagonally across the corners of the square and an octagon or " eight square " figure will be the result. This method is nearly correct but is not exact.

Example. Lay out an octagon on a stick that is 12″ square.

FIG. 87.

FIG. 88.

Explanation. This example is illustrated in Fig. 89. Find the center of the stick by drawing diagonals from corner to corner and scribe the center line, using a try-square. This will give the lines *A B* and *C D* shown in the figure. The stick is 12″ square; therefore, you should take twelve divisions off of the scale. Lay this distance off in each direction to the right and left of the points *A*, *B*, *C* and *D*.

FIG. 89.—Laying out an Octagon.

Join these points with straight lines across the corners. The result will be an octagon.

166. The Essex Board Measure. The Essex board measure is used to determine the number of board feet contained in a stick without figuring it out. This scale is

shown along the center of the body in Fig. 90. The starting point for each determination is the figure 12 on the outer

edge. This is the number of board feet in a 1″ board 12″ wide and 12 feet long. The small figures in a line under the 12 mark represent other lengths of boards. To find the number of board feet in a piece which is more or less than 12″ wide and from 8 to 15 ft. long, run down the column under the 12 mark until the proper length is found. Then run along either to the right or left until under a mark on the outer edge which corresponds to the proper width of the board and the small figure will be the required value in board feet.

Since a board 12 feet long would have as many board feet as it is inches wide, the figures are omitted for this length. Since a board 6 feet long would have one-half as many board feet as it is inches wide, this length is also omitted. If a board is less than 8 feet long its board measure may be found by dividing its double by two. If a board is longer than the lengths given on the scale, divide the length into two parts, find the board measure for each of these and add them together. Don't try to

Fig. 90.

work out a problem on the Essex board measure if you can do it quicker in your head.

Example. How many board feet in a board 1″ thick, 15 feet long and 8″ wide?

Explanation. Referring to Fig. 90 under the figure 12 on the outer edge look down until you find 15, which is the length of the board. Then run between the lines to the left until you are under the figure 8 on the outer edge of the scale. This represents the width of the board. Here you will find the figure 10. There are 10 feet B.M. in the board.

Example. How many board feet in a board 2″ thick, 22 feet long and 10″ wide?

Explanation. Divide the length into two parts, one 10 ft. and one 12 ft. We know at once that a board 10″ wide and 12 ft. long will have 10 ft. B.M. Since this board is 2″ thick this part will contain 20 ft. B.M. For the part 10 ft. long look down under the figure 12 to find 10 and to the left until you come to the figure 10 on the outer edge of the body. Here read 8-4 which means $8\frac{4}{12}$ or $8\frac{1}{3}$ ft. B.M. Since the board is 2″ thick double this to get $16\frac{2}{3}$. Adding this to the value 20, previously found in the first part, we get $36\frac{2}{3}$ ft. B.M. in the piece.

167. The Diagonal Scale. Fig. 91 shows the diagonal scale used on some steel squares. Its purpose is to enable the workman to make measurements to the hundredth of an inch. A square one inch on a side is divided into ten equal parts on each side. In one direction

FIG. 91.

these lines are joined with lines parallel to the sides of the square. In the other direction the point marked *O* is

joined to the point
1 on the opposite
side. You will now
see that the dis-
tance on the line
$b1$ from the zero
vertical line to the

diagonal intersection is $\frac{1}{100}$th of an inch.
In like manner the distance from the zero
vertical line to the diagonal intersection
on $h7$ is $\frac{7}{100}$ths of an inch.   To read
.78″ on the scale, using a pair of dividers
you place one point on the intersection of
the zero vertical line with the horizontal
line $i8$. The other divider point should
be extended to touch the diagonal in-
tersection on $i8$ just beyond the vertical
line marked 7. The distance between the
divider points will then be $\frac{70}{100} + \frac{8}{100} =$
$\frac{78}{100} = .78$, which is the required distance.
Read tenths on the horizontal scale and
hundredths on the vertical scale.

168. The Polygon Scale. This scale
is useful for finding the correct angle to
cut sticks of lumber so that they will
form regular polygons when fitted to-
gether. It is illustrated in Fig. 92.   On
some squares this scale is placed along the
center of the face of the tongue. The
first figure indicates the number of sides
in the polygon. The second figure indi-
cates the point on the body, and the third
figure the point on the tongue that is

Fig. 92.

made to coincide with the edge of the stick in order to get
the correct angle as shown in Fig. 93. If it is desired to
cut a polygon of eight sides you would take 18 on the
body and $7\frac{1}{2}$ on the tongue of the square. Mark the
stick along the edge of the tongue as illustrated in the
figure. Eight pieces of equal length cut to this angle will
fit together to make an eight-sided figure.

169. Geometric Applications. If the workman under-
stands fully the properties of the right triangle and the
applications of geometry, he will have but little trouble

FIG. 93.—Getting Correct Angle for a Polygon Frame.

in making his steel square serve him as a very useful tool.
A few of the common applications are given here. Others
will occur to the practical man in connection with his daily
work.

170. Hypotenuse of a Right-angled Triangle. To find
the length of the hypotenuse of a right-angled triangle
when the base and altitude are unequal, lay a two-foot
rule across the square with the zero mark of the rule at the
point on the body which corresponds to one leg of the tri-
angle and the edge of the rule crossing the tongue at a point
corresponding to the other leg of the triangle. The
reading of the rule will give the length of the hypotenuse.

Example. What is the length of the hypotenuse of a
triangle the base of which is 8 ft. and the altitude 14 ft.?

Explanation. In this case it is convenient to allow 1″ on the rule and on the square to represent a foot in the triangle. Referring to Fig. 94, we place the zero mark of the rule on the 8″ mark on the body and lay the edge of the rule across the 14″ mark on the tongue. At this point the rule reads $16\frac{1}{8}$″. This means that the hypotenuse is $16\frac{1}{8}$ ft. long or, expressing this in feet and inches, 16′ $1\frac{1}{2}$″.

Fig. 94.—Finding Length of Hypotenuse.

171. To Divide a Line. Suppose we wish to divide the width of a board into ten equal parts for ripping into strips. Lay the square diagonally across the board so that the zero mark is at one edge of the board and the 10″ mark is at the other edge as shown in Fig. 95. Then make a mark exactly at each one of the inch divisions on the edge of the square. Drawing a line parallel to the edge of the board through each one of these points will divide the board into ten equal strips.

172. To Bisect an Angle. Measure out equal distances

on each leg of the angle, OA and OB in Fig. 96. Hold the
square so that the same figure on each side of the square

Fig. 95.—Finding Width of Strips.

coincides with the points A and B. The vertex or point
of the square will then be at the exact middle of the angle.

Fig. 96.—Bisecting an Angle.

Mark this point D and join it with a straight line to the
vertex of the angle O. The line OD is the required bisector
of the angle.

Summary of Chapter XV

93. The brace measure is used to find the length of a diagonal brace.   (Sec. 164.)

94. The octagonal scale is used to find the correct angle for dressing a square stick into an octagon.   (Sec. 165.)

95. The Essex board measure scale is used to find the number of feet B.M. in any board.   (Sec. 166.)

96. The diagonal or hundredth scale is used to measure hundredths of an inch.   (Sec. 167.)

97. The polygon scale is used to find the correct angle to cut sticks of equal length to form polygons having from five to twelve sides.   (Sec. 168.)

98. The steel square may be used to find the length of the hypotenuse of any right-angled triangle.   (Sec. 170.)

99. The steel square may be used to divide a given line into any number of equal parts.   (Sec. 171.)

100. The steel square may be used to bisect an angle. (Sec. 172.)

PROBLEMS

220. Find the length of a diagonal brace for a square 54″ on a side.

221. Find the length of a diagonal brace for a square 4′ 9″ on a side.

222. Find the length of a diagonal brace for a square 10′ on a side.

223. Draw a diagram showing how to lay out an octagon on a stick 6″ square, using the octagon scale.   If you take your work from Fig. 87 you will have to make your drawing half size.

224. Using the Essex board measure scale find how many board feet there are in a board 2″ thick, 9″ wide and 16′ long.

225. How many feet B.M. are there in 16 pieces 1½″×10″, 26′ long?

226. How many feet B.M. are there in a board 1″×14″, 18′ long?

227. A six-sided figure each side of which is 18″ long is required. Tell how to find the correct angles to cut the boards.

228. What is the length of the hypotenuse of a triangle the base of which is 6 ft. long and the altitude 10 ft.?

229. What is the length of the hypotenuse of a right-angled triangle 16″ on a side?

230. Divide a line which is 6″ long accurately into 13 equal parts.

231. Construct a diagonal scale similar to the one shown in Fig. 91.   Show the distance .78″ on your diagram.

232. Bisect a 45° angle, using a steel square.

CHAPTER XVI

USE OF THE CARPENTER'S SQUARE IN FRAMING.
FLOOR BRIDGING. STAIR BEAMS. RAFTER
FRAMING. ROOF PITCHES. RAFTER TABLE.
ROOF FRAMING. HIP OR VALLEY RAFTERS. JACK
RAFTERS

NOT only may the steel square be used for the purposes
indicated in the last chapter, but it may also be used very
conveniently in the processes required for framing a building.
As everyone knows, it may readily be used for determining

FIG. 97.—Cuts for Floor Bridging.

a right angle. It may also be used to determine many of
the other angles to which it is necessary to cut the lumber
in framing. These angles are called **cuts or bevels.**

173. To Find the Cuts for Floor Bridging. Take the
width of the joist on the tongue of the square and the dis-
tance between the joist on the body. Take the measure-
ments on opposite sides of the stick as shown in Fig. 97.
The tongue will give the cut and the body will give the
length.

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174. To Find the Cuts on a Stair Beam. Suppose the stair is to be laid out with $7\frac{1}{2}''$ risers and $9''$ treads.   Then it is simply necessary to lay the square on the beam in such a manner that the reading on the tongue from the edge of the beam is $7\frac{1}{2}''$ and a similar reading on the body is $9''$, as shown in Fig. 98.   Mark the line and repeat for each

Fig. 98.—Laying out Cuts on a Stair Beam.

succeeding step.   Remember that the thickness of the floor must be taken into account when laying out the top and bottom steps.

175. Rafter Framing.   Before taking up the subject of roof framing it is necessary to explain some of the terms and customs commonly used.   The **span** of the roof is the same as the width of the building as shown in Fig. 99.   The **run** of the rafter is one-half of the span and the **rise** is the vertical distance from the top of the plate to the top of

the ridge. The **pitch** of the rafter is the ratio of the rise to the span. The **rafter length** is the distance from the outside corner of the plate to the center of the ridge. The **extension** is the distance from the outside corner of the plate to the end of the rafter. When finding the actual length of a rafter, the distance from the outer corner of the plate to the center of the ridge is first found. The square is then set for

Fig. 99.—Span of a Roof.

the ridge cut as explained in Sec. 177 and one-half the width of the ridge subtracted in the direction of the body of the square. Sufficient length must then be added for the extension. The result will be the exact length of the rafter.

176. Pitch. The most common pitches are indicated in the table. They can be found on the steel square by laying a straightedge across the square with one end on the figure 12 on the body and the other end on that figure of the tongue which corresponds to the rise as illustrated in Fig. 100.

TABLE XVI

COMMON ROOF PITCHES

$\frac{1}{6}$ pitch is	12 on the body and	4 on the tongue.
$\frac{1}{4}$	12	6
$\frac{1}{3}$	12	8
$\frac{5}{12}$	12	10
$\frac{1}{2}$	12	12
$\frac{5}{8}$	12	15
$\frac{3}{4}$	12	18

FIG. 100.—Finding Pitch.

177. To Find the Cuts for a Common Rafter. The cut at the top of the rafter is called the **ridge cut** or the **plumb cut.** The cut at the lower end of the rafter is called the **bottom cut** or **heel cut.**

First, to find the plumb cut place the square as shown in the upper position of Fig. 101. Use 12 on the body and other figures on the tongue, depending upon the pitch of the roof. For example, if the rafter is to be set at one-third pitch use 12 on the body and 8 on the tongue as indicated in the table above. For pitches not given in the table

multiply the pitch by 2, express as a proportion and solve
for the tongue reading: Thus, for one-fifth pitch

$$\frac{1}{5} \times 2 = \frac{2}{5}$$
$$2 : 5 :: x : 12$$
$$x = 24 \div 5 = 4\frac{4}{5}. \qquad Ans.$$

Next, to find the heel cut place the square as shown in
the lower portion of Fig. 101. If the rafter is to extend

Fig. 101.—Finding Rafter Cuts.

beyond the plate, the square is again set so as to give the
plumb cut indicated in the figure. The process is repeated
again for the end cut.

178. The Rafter Table. The rafter table found on the
back of the body of **rafter squares** or **framing squares** is used
to find the length of a common rafter for various pitches.
This table is shown in Fig. 102. The graduations on the
outside edge of the body are used in connection with the

table. These graduations are in twelfths and the inches may be used to represent either inches or feet while the twelfth marks represent either twelfths of a foot (inches) or twelfths of an inch. These figures on the edge when making use of the table correspond to the run of the rafter. To use the rafter table, look at the left end of the table to find the required pitch. Then under the mark on the outer edge which corresponds to the run of the rafter in feet will be found the required length in feet, inches and twelfths of an inch. Thus a rafter set at one-half pitch with a run of ten feet is $14'\ 1\frac{8}{12}''$ long. Again a rafter set at quarter pitch with a run of 12 ft. is 13' 5'' long.

When the run is given in inches the length will be in inches. Thus, a rafter set at quarter pitch with a run of 8'' would be $8\frac{1}{12}'' + \frac{4}{12}$ of $\frac{1}{12}''$, which, neglecting the added fraction, is approximately $8\frac{1}{12}''$. If the run is given in both feet and inches find each separately and add the lengths together.

Example. What is the length of a common rafter set at one-third pitch with a run of 10 ft. 6 in.?

Fig. 102.

For run of 10 ft. the rafter length is 12' $0\frac{3}{1\frac{1}{2}}$''
For run of 6'' the rafter length is $7\frac{2}{1\frac{1}{2}}$''

_____

Total length   12' $7\frac{5}{1\frac{1}{2}}$'' *Ans.*

Explanation.   Looking for the run in feet, we first find that the length for a 10 ft. run will be 12' $0\frac{3}{1\frac{1}{2}}$''.   Now finding the length for the run in inches we get $7\frac{2}{1\frac{1}{2}}$'', disregarding the last figure in the table.   Adding these figures together gives 12' $7\frac{5}{1\frac{1}{2}}$'' for the distance from the outer edge of the plate to the center of the ridge.   For the actual length deduct one-half the thickness of the ridge and add sufficient for the extension.

179. Roof Framing.   Up to this point we have taken up only the lengths and cuts for common rafters.   Any roof which is framed for gables, hips and valleys will require rafters of various lengths having both plumb cuts and side cuts.   Fig. 103 shows the framing for such a roof.   The **ridge board** marked *A* is the piece at the center to which the rafters are nailed.   The **plate** marked *B* is the board placed on top of the studs to which the foot of each rafter is secured.   The **common rafters,** *C*, are those which set square against the ridge board and which have no side cut.   **Hip rafters** are those marked *D*.   **Valley rafters** are those marked *E*.   **Jack rafters,** marked *F*, are those the upper end of which set against the hip rafter.   **Cripple rafters**—rafters having no foot—span the space between the ridge board and valley rafter or between hip and valley rafters.   They are marked *G* in the illustration.

180. Length of Hip or Valley Rafters.   The length of the hip rafter is the hypotenuse of a right-angled triangle, one side of which is the common rafter and the other side is the distance on the plate from the foot of the hip rafter to

FIG. 103.—Framing for a Roof.

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the foot of the common rafter. The length of the valley rafter is also the hypotenuse of a similar triangle, and for this reason the following example and explanation will apply to a valley rafter as well.

Fig. 104.—Hip Rafter.

Example. What is the length of the hip rafter shown in Fig. 104?

$$5'\ 8'' = 5.66'$$
$$5.66^2 = 32$$
$$4^2 = 16$$
$$16 + 32 = 48$$
$$\sqrt{48} = 6.93$$
$$6.93' = 6'\ 11\tfrac{3}{16}'' \qquad Ans.$$

Explanation. The length of the common rafter having 4 ft. run for one-half pitch roof is 5' 8''. The distance from the foot of the hip rafter to the foot of the common rafter is 4'. The hip rafter is the hypotenuse of this triangle. Solving by finding the square root of the sum of the squares of the base and altitude gives 6' $11\frac{3}{16}''$ for the length of this rafter. Notice that the hip rafter is not shown in its true length in the figure.

181. Cuts for Hip or Valley Rafters. The rise for a hip or valley rafter is the same as the rise for the common rafter. The run of a hip rafter is the horizontal distance from the outside corner of the plate at the foot to a vertical line from the center of the ridge against which it rests. This is equivalent to the length of the corresponding common rafter. In every case the ratio of the run for the hip or valley rafter to the run of the common rafter is as 17 is to 12. To find the plumb and heel cuts for a hip or valley rafter for a given roof pitch we proceed exactly as for a common rafter, but we use the 17 mark on the body instead of the 12 mark.

Example. What figures should be used in marking the plumb and heel cuts for the hip rafter of a roof of one-third pitch when the run of the common rafter is 8 ft.?

Explanation. To find the plumb and heel cut of a common rafter having one-third pitch use 12 on the body and 8 on the tongue as explained in Sec. **177.** For the hip rafter use 17 on the body and 8 on the tongue.

182. Hip Rafter Table. The necessary information for finding the side cuts for hip and valley rafters and the length and side cuts for jack rafters may be found in the hip rafter table shown in Fig. 105.

In the table under the heading PITCH is given the reading for the pitches in common use. The fractions represent

the pitches and the figures to the right are the ones used for determining the plumb and heel cuts of common rafters.

The next set of figures under the heading HIP indicate the length of the hip and valley rafters for each foot of run for the common rafter of the different pitches. To show how this is used let us take the same example that is worked out above.

Example. What is the length of the hip rafter shown in Fig. 104?

$$1-8-10\times4=4-32-40$$

$$4-32-40=4'\ 32\tfrac{10}{12}''=6'\ 11\tfrac{4}{12}''=6'\ 11\tfrac{1}{3}''$$

$$Ans.$$

Explanation. From the table for every foot run of the common rafter the hip rafter is $1'\ 8\tfrac{10}{12}''$ long. Since the run of the common rafter is 4' we multiply this by 4 and reducing the product to its simplest form find the length of the hip rafter to be $6'\ 11\tfrac{1}{3}''$, which is approximately the same value that we got by solving the triangle.

183. Jack Rafters. The next set of figures on the hip rafter table gives the length of the shortest jack rafter when they are spaced 16″ on center. Since

Fig. 105.

these rafters are evenly spaced the second will be twice as long as the first and the third three times as long as the first. The lengths for 24" spacing are also given.

Example. Find the length of the third jack rafter set at one-half pitch.

$$22\tfrac{5}{8}'' \times 3 = 67\tfrac{7}{8}'' = 5'\ 7\tfrac{7}{8}'' \qquad Ans.$$

Explanation. The first jack rafter set 16" on centers at one-half pitch will be $22\tfrac{5}{8}''$ long as indicated in the table. The third rafter will be three times as long as this or $5'\ 7\tfrac{7}{8}''$.

The plumb and heel cuts for jack rafters are the same as for common rafters of the same pitch. To find the side cut use the next set of figures to the left in the table. For one-half pitch take 17 on the body and 12 on the tongue and and mark the rafter as shown in Fig. 106.

Fig. 106.—Side Cut for Rafter.

184. Side Cut for Hip Rafters. To find the side cut for hip or valley rafters use the figures under the heading HIP marked SIDE-CUT. For one-half pitch the setting on the square will be 8 on the body and 11 on the tongue. Mark the cut along the tongue.

185. Length of Hip or Valley Rafters. The remainder of the table indicates the length of the hip or valley rafters for the run of the corresponding common rafter for the

various pitches.   Find the run of the common rafter on the outside edge of the scale.   Under this figure will be found the length of the hip or valley rafter for the corresponding pitch.

Example.   Find the length of hip rafter required for one-half pitch roof when the building is 24 ft. wide.

Explanation.   The run of the common rafter is one-half the width of the building.   Under the figure 12 on the outer edge and to the left between the lines which represent one-half pitch find the figures $20-9-5$.   This means that the hip rafter is $20' \ 9\frac{5}{12}''$ long.

186. Cuts for Cripple Rafters.   The plumb and side cuts for cripple rafters are the same as for jack rafters.   The cuts at the bottom of a cripple rafter are the same as those at the top.

187. Actual Lengths.   In all of the above tables the lengths given are from the outside edge of the plate to the center of the ridge or hip.   To find the actual length deduct one-half the width of the ridge or hip and add sufficient material for the extension at the eaves.

Summary of Chapter XVI

101. The steel square may be used to find the cuts for floor bridging as explained in Sec. 173.

102. It may be used to find the cuts on a stair beam. (Sec. 174.)

103. It may be used to find the cuts on a common rafter. (Sec. 177.)

Fig. 107.—Struts.

104. The rafter table may be used to find the length of common rafters required for various runs and pitches. (Sec. 178.)

105. The length of a hip or valley rafter is equal to the hypotenuse of a right-angled triangle, the base of which is the distance from the foot of the hip rafter to the foot of the common rafter and the altitude of which is equal to the length of the common rafter. (Sec. 180.)

106. The plumb and heel cuts for hip and valley rafters may be found by using 17 on the body instead of 12 as explained in Sec. 181.

107. **The hip rafter table may be used** to find the plumb and heel cuts of common, hip and jack rafters; to find the side cuts of these rafters, and to find the length of hip, valley and jack rafters. (Sec. 182.)

FIG. 108.—Roof Framing.

108. **The cuts for cripple rafters** are the same as for jack rafters except that the bottom cuts are like the top cuts. (Sec. 186.)

109. **The actual length of the rafters** is found by deducting one-half of the ridge or hip and adding sufficient material for the overhang. (Sec. 187.)

PROBLEMS

233. Ten inch floor joists are placed 16″ on centers. Draw a diagram to scale showing how to find the top and bottom cuts for the bridging.

Fig. 109.—Plan of Roof.

234. A stair joins two floors, the second of which is 10′ 3″ above the first. Twice the riser plus the tread should about equal 24″. Make a sketch showing how to lay out the cuts on the stair beam.

235. A building is 24′ wide. The roof ridge is 8′ above the plate. What is the pitch of the roof?

236. A building is 32′ wide. The roof ridge is 16′ above the plate. How would you determine the plumb and heel cuts for the common rafters?

237. The roof of a building has a pitch of three-eighths. How would you find the plumb and heel cuts of the common rafters?

238. What is the length of a common rafter set at five-twelfths pitch when the run is 12′ 6″?

239. What would be the length of the hip rafters for the roof in the problem above?

240. What is the length of a valley rafter for a roof of one-half pitch when the span is 16 ft.

241. Show how to find the cuts for the struts marked a and l in Fig. 107.

HINT.—Struts b are perpendicular to the rafters.

242. By using the hip rafter table wherever possible find the number of lineal feet 2×6, 2×4 and 4×4 required for the roof framing in Fig. 108.

NOTE.—The pitch of the lower section is not to be found in the rafter table.

243. Show how to find the side cuts for the jack rafters shown in Fig. 108.

244. Show how to find the side cuts for the hip rafters shown in Fig. 108.

245. Using the hip rafter table, find the length of a hip rafter for a roof having one-fourth pitch when the building is 18 ft. wide.

246. Find the actual lengths of all the rafters in the roof shown in Fig. 109. Make a roof framing plan to accompany your solution. Space the rafters 16″ on centers.

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