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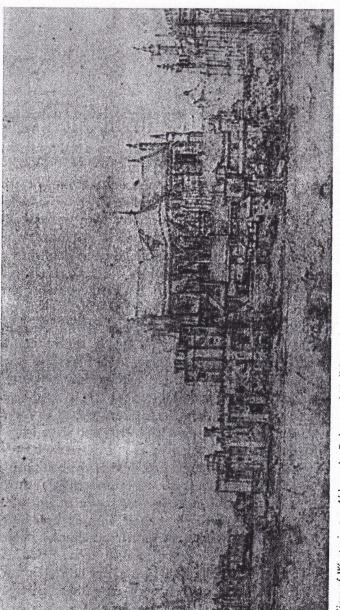
An Apsidal Timber Roof at Westminster

Introduction

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It is only recently that comprehensive detailed studies have been made of timber roofs in England. As late as 1924, for example, the Royal Commission on Historical Monuments devoted a whole volume to Westminster Abbey¹ without a single mention of the timber roofs, despite the fact that these were constructed at the same time as the stone vaults beneath. This lack of interest contrasts with active studies on the Continent of Europe, and notably in France, where descriptive histories of *charpentes* have been published for over 300 years². Thus McDowall *et al*³, who studied the roofs at Wesminster Abbey, found it difficult to relate these to those of other major churches, and to assess their place in the development of English roof construction.

By the 1960's, at which time a major restoration was under way, the roofs at Westminster Abbey were in urgent need of repair, and had become extraordinarily cluttered with subsidiary trusses and shores. The roofs to the nave and both transepts have now been reconstructed to the original design of trussed rafters with collars and scissor braces. The thirteenth-century roof over the presbytery, however, has been largely carefully restored, with later additions removed. It has great interest for the architectural historian and is, as it happens, of great interest to the engineer, and for the same reason: it covers a choir with an apse. Westminster Abbey is, of course, a French church, and it has a chevet rather than the square English termination. The timber roof is now the oldest surviving roof in England with an apsidal end4; this semi-circular, or rather, polygonal, croupe posed engineering problems which were not in fact solved by the designer, and the roof has experienced unusual deformations in addition to the usual distortions of timber frameworks.



View of Westminster Abbey, the Palace and Hall from the east. Wenceslaus Hollar, c. 1630.

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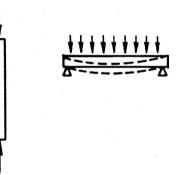
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The deformations which arise in timber roofs are natural consequences of the forces which they are required to carry, and this essay attempts to describe these forces and their modes of action. To this end the main portion of the essay discusses the general problem of the framing of a timber roof; some detailed conclusions are then drawn for Westminster. In particular, an insight may be gained into the nature of the active structural forces which must, of necessity, govern the design of any restoration work for these and similar structures.

The structural action of timber roofs

Wood is a material which can resist tension as well as compression (unlike stone, which is weak in tension, and should be used as far as possible solely to resist compressive forces). Thus a baulk of timber can be used effectively as a prop in endlong compression; equally, it can be used in bending under transverse loading, in which case the underside of the baulk works in tension.



In pure compression (as a prop) the axial shortening of the timber is very small. A rafter, 8 inches square and 36 ft. long, might shorten by about 10 thousandths of an inch under an axial load of 2000 lb. The same rafter used as a beam to span 36 ft, and carrying a distributed tranverse load of 2000 lb would, however, deflect some 4 inches at the centre. Moreover, the stresses induced in the timber by the two types of loading have very different values. In the first

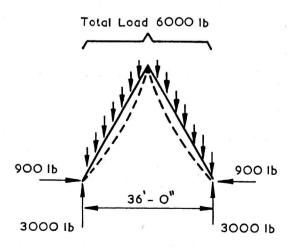
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case, the direct compressive stress is about 30 lb/in², whereas the bending stress for the laterally-loaded beam is dangerously high at some 1250 lb/in² (this last figure is a significant fraction of the crushing strength of oak).

Thus, to be structurally efficient, timber members should work primarily in compression (or, of course, in pure tension), and bending action should be avoided. Not only will deflexions then be very much smaller, so that the whole structure is *stiffer*; stresses will also be markedly reduced, so that the structure is *stronger*. If the roofs at Westminster are viewed in this way, then the purpose of the various component members becomes clear.

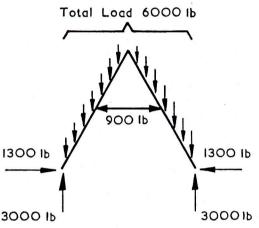
To start with the unbraced rafters, suppose that two 36 ft long members, 8 inches square in section, are pinned together to form a simple couple roof. If the rafters are spaced at about 2 ft or 2 ft 6 in from the next pair, then the loading on an individual frame may be estimated; the total load acting on one pair of rafters, due to the lead sheeting, timber battens, and self-weight of the rafters themselves will be taken.as 6000 lb. The rafters will be connected to timber wall plates, and these must provide a vertical reaction of 3000 lb at the foot of each rafter. In additon, each rafter foot thrusts out with force of about 900 lb, which must be resisted by a horizontal reaction from the wall plate and eventually by the masonry walls and buttresses.



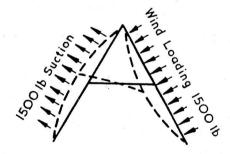
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The inward bowing at mid-height of the rafters is about 3 inches, and the maximum bending stress is about 1000 lb/in^2 . Both these figures are substantially reduced by the insertion of a collar beam at mid-height of the rafters. The compressive force in the collar beam itself may be estimated as 900 lb (the rafters carrying the same uniformly distributed load of 6000 lb as before). The horizontal thrust at the rafter feet is *increased* to about 1300 lb. The maximum deflexion of the rafters between the apex and the collar beam (or between the collar beam and the feet) is reduced to under 1/4 inch, and the greatest bending stress is about 250 lb/in².



Both these figures might be acceptable as working values, and the primary triangulation of the roof has resulted in a satisfactory structure as far as vertical load is concerned. However, the roof is still very weak when considered as a structure required to resist wind loading. The collar beam



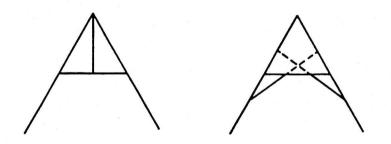
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will offer almost no stiffening to the rafters under the usual action of wind, and each rafter might again deflect a total of about 3 inches (at collar-beam level) in a strong gale. The reason for knee braces is now apparent; these are continued as full scissor beams at Westminster. The braces help the structure to resist the action of wind, and they also reduce still further any remaining tendency of the rafters to bend under the vertical dead load.

It may be noted here that the addition of a king post to a truss consisting only of rafters and collar beam is merely a kind of secondary triangulation. This may be of help in



the construction of the roof, but, in the final state of the truss, the king post itself is virtually incapable of carrying vertical load. This is because the king post rests at its lower end only on the collar beam, and any load transmitted by the king post must be resisted by *bending* of the collar beam. Since a timber member is so flexible in bending, it will at once deflect beneath the attempted loading, and the king post will be relieved of all but a very small residual thrust.

The conclusion drawn from this particular example of the king post may be broadened to apply to more complex roof systems. In practice, a timber roof is highly redundant (in the technical sense); that is, there are several alternative ways in which the members may be stressed in order to carry a given loading. So long as there is the possibility of any individual member working both in axial thrust and in bending, then even if the roof is properly triangulated, there will always be some room for discussion as to the precise way the truss acts as a whole. However, since the members are so m thrust, th bending selves alc So far, roof of t or witho tural for bers. All and the also be si The ac effect on will not the rafte tem, and problem tinuous t slowly g the incre rafters a ment mi any mar deforme

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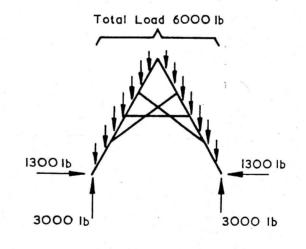
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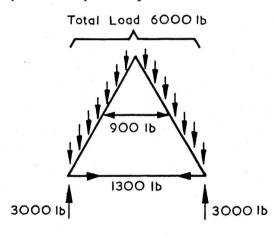
are so much more flexible in bending than they are in pure thrust, they will tend to relieve themselves, if they can, of bending stress, and the structural forces will align themselves along the individual members.

So far, then, there is some certainty that an undeformed roof of the type considered, with a collar beam and with or without full scissor-beam braces, will have primary structural forces acting more or less along the lines of the members. All these members will be comfortably in compression, and the corresponding stresses will be low. Deflexions will also be small.

The addition of further secondary members will have little effect on the main force system. For example, ashlar pieces will not much reduce the horizontal thrust at the feet of the rafters; the thrust is an essential feature of the roof system, and the resisting of this force raises serious structural problems. A masonry wall subjected to such a steady continuous thrust will, unless extraordinary measures are taken, slowly give way. The roof truss will accommodate itself to the increased span by bending of the rafters, but since the rafters are so flexible in bending, considerable lateral movement must occur before the horizontal thrust is relieved to any marked extent. At this stage the rafters will be grossly deformed.



A tie between the feet of the rafters would, of course, lead at once to a self-equilibrating roof system, the walls thereby being relieved of all necessity to provide horizontal thrust. It is easy enough in modern times to provide a light metal tie between rafter feet, but the design of a timber tie is not so simple. This is almost entirely due to the difficulty of constructing suitable joints between wooden members that can transmit tensile forces. A simple butt joint, for example, perhaps notched and with or without a pinned tenon, will be perfectly satisfactory in compression but weak in tension.



The problem is even more difficult if ties are not provided to each pair of rafters, but only at intervals. In this form of construction, which was used at Westminster and is quite common, reliance is placed on the wall plates to pick up the horizontal thrusts from the rafters and to transmit these thrusts to the more widely spaced ties. Any weakness in design of the ties, or decay of the timber, will lead to the horizontal thrusts being imposed on the walls rather than absorbed in the ties.

In the whole of this discussion the rafter system has been viewed two-dimensionally; it has been assumed that all forces act in the plane of the truss, that is, in a plane perpendicular to the axis of the nave or transept, as the case may be. This may well be a reasonable assumption; longitudinal forces will certainly be small, for example, in a roof running t longitudi be stiffen overall st ways, for onal brac positive roof does Thus, i be such compress in this w the scant portant. exact ma is one of The n connexio trusses n whole ro-Finally rest of th the supp importan of the ho

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running between towers or gable walls. Even if there are no longitudinal forces, however, a properly-designed roof must be stiffened in the longitudinal direction in order to achieve overall stability. Such stiffening could be provided in several ways, for example by ridge beams and purlins, or by diagonal braces in the plane of the rafters. As will be seen, more positive provision for longitudinal forces is required if a roof does not terminate in a flat gable.

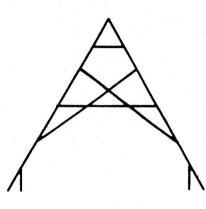
Thus, in summary, the basic triangulation of a roof should be such that all members work as far as possible in pure compression, bending being avoided. If the design is made in this way, stresses will be very low and, correspondingly, the scantlings of the various members are relatively unimportant. Equally, there is no need to know accurately the exact magnitudes of the applied loads. The first problem is one of devising a satisfactory geometry for the truss.

The next task is the practical one of providing suitable connexions between the members, and, thirdly, the roof trusses must be braced together in some way so that the whole roof has a measure of longitudinal stability.

Finally, the completed design must be mounted on the rest of the fabric, and particular attention must be given to the support of the roof truss on the masonry walls. The allimportant problem here is that of the satisfactory resistance of the horizontal thrust of the rafters.

The 13th Century roofs at Westminster Abbey

McDowall *et al* describe the Abbey roofs as they found them in 1964. At that time the nave roof and that of the south transept had been reconstructed, but the roofs of the presbytery and of the north transept, although heavily restored 250 years earlier, were essentially the mediaeval originals. Each roof truss, spaced at between 2 ft and 2 ft 6 in. from the next, was to an apparently identical pattern, and consisted of a pair of rafters (of scantlings approximating those used in the illustrative calculations above), a main collar beam and an upper collar, full scissor braces, and ashlar



pieces. A tie beam was originally provided for every seventh or eighth truss, the inner wall plate being tenoned into the ties. These ties were clearly quite inadequate, since in most of the frames the feet had spread with consequent gross bowing of the rafters.

The scissors beams are halved together where they cross, and they are also halved to the collar beam. There is a curiously elaborate notched and tongued joint between the foot of the scissor brace and the rafter.

No provision appears to have been made for the general longitudinal stability of the trusses, although some rudimentary longitudinal braces were once present in the roof to the north transept. The roofs to the two transepts and to the nave are terminated by flat masonry walls, and there are no out-of-balance longitudinal forces to be carried. The trusses were in fact connected together by the horizontal battens carrying the roofing; these battens would offer virtually no resistance to movements of the roof as a whole in the longitundinal direction, but would at least ensure that all the trusses swayed out of the vertical together. Since in fact the end trusses would be restrained by the flat gables, the three roofs would achieve some stability in this way.

The basic triangulation of the roof trusses is good, and the first problem of design, that of devising a satisfactory geometry, was solved. Physical construction, however, was not so good. The halved joints in the scissor beams, for example, are not easy to make, and reduce the section of the mei mary de source joints a as well It is structur The fai most g carpent sketch would would r dimensi it indica be place firmed 1235. \ tenons of cons shown. their or a physic

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the members substantially; although strength is not a primary design requirement, each halved joint is a potential source of weakness. Similarly the notched and tongued joints at the feet of the scissor beams are weak in design as well as being difficult to execute.

It is when the roofs are viewed as three-dimensional structures, however, that their real weaknesses are revealed. The failure to provide proper longitundinal stiffening almost gives the impression of an unsophisticated master carpenter trying to construct a whole roof system from the sketch of a single two-dimensional truss. Such a drawing would dictate the overall configuration of the roof, but would not in itself give any guidance as to how the threedimensional framing should be realized in detail; nor would it indicate how and where ties between the rafter feet should be placed. (That such sketches were made as patterns is confirmed by the sketch book5 of Villard de Honnecourt, c. 1235. Villard shows several roof trusses in which pinned tenons and notches are just indicated, but no other details of construction are given, and no longitudinal framing is shown. Users of such sketches would have had to rely on their own practical experience to translate the design into a physical reality.)

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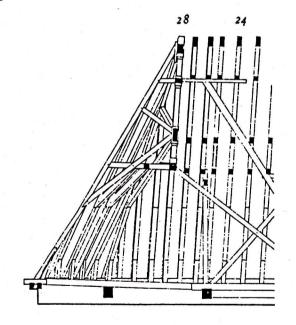
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These remarks apply even more forcibly to the framing of the presbytery roof, where the main trusses are of the type already described. Quite apart from the stabilizing of these trusses, it would seem that the apsidal end was not considered as a separate structural problem; the framing of the *croupe* is exactly the same as the framing elsewhere. Essentially, each truss in the *croupe* is a standard truss cut in half, and framed in towards the centre of the last plane truss (the *ferme-maitresse*, numbered No. 28 by McDowall *et al*); this last truss is provided with a king post to receive the half collars and scissor beams. Thus the whole of the roughly semicircular end of the roof is centred on the king post of truss 28.



Now the trusses must indeed be cut in half to form the *croupe*, but they must also be propped horizontally at the apex and at the collar beam if they are to remain in position. The vertical load on a single half frame at Westminster might be about 2000 lb, and the required propping force at collar-beam level is then 600 lb, with half this value at the apex. All the half frames together will com-

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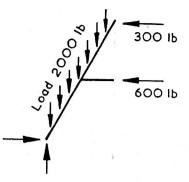
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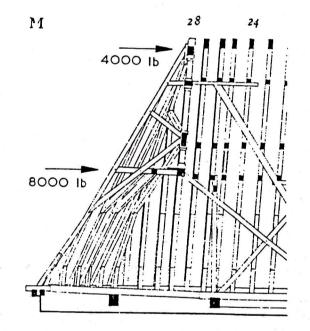
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bine to thrust westward with a force of about 8000 lb at collar-beam level, with again about half this thrust at the apex. The half frames are in fact all leaning against truss 28, which must somehow be supported by the rest of the presbytery roof.

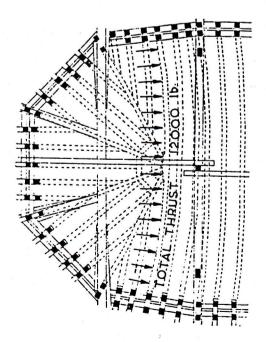


In the original structure, the force of 8000 lb could only be resisted by lateral bending of the unsupported collar beam of truss 28. This collar beam, however, is a member suited only to carrying axial load, and it is not surprising

to find that it was broken right through, at a section weakened by halving.

Similarly, the scissor beams should come into play under wind loading on the roof. The scissor beams are framed into the king post, a member which in this case can resist neither lateral nor vertical forces.

The whole of the framing of the apse roof is, in fact, basically unsound. The king post, in particular, is a ludicrous attempt to solve a structural problem (whose existence, of course, was almost certainly not suspected); it may be unique in English mediaeval roofs, but it is neither *bello* nor *lodevole* (to use the terms of the Milan expertises at the end of the fourteenth century⁶). There seems to have been no appreciation at all at Westminster that large longitudinal forces must arise from the framing of the *croupe*. That this was local ignorance is demonstrated by the almost contemporary roof of Notre-Dame, Paris, where elaborate provision was made to prop the *ferme-muitresse* and to absorb and distribute the out-of-balance thrust.



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The out-of-balance forces which broke the collar beam of truss 28 at Westminster cannot therefore have been carried by that collar beam, but must have been transmitted to the west by some other means. The only medium available for this transmission is again the sheeting battens, brought into play by the tendency of the apse roof to sag westward. The half collar beams of the apsidal trusses would have been inoperative, and the total out-of-balance force of say 12000 lb would have been transferred more or less uniformly by the battens to the surface of the rafters of truss 28.

As has been noted, the horizontal battens of the main presbytery roof would provide no resistance to this longitudinal force. So long as the presbytery roof was continuous with the nave roof, there may have been a chance of the trusses remaining roughly vertical. There is now little evidence to indicate the date at which all the trusses were actually forced 5° out of the perpendicular (a sway of about 3 ft), but it could have been in 1725 when it seems the roof was cut to admit the central tower.

Conclusions

The implications of this simple study of the mechanics of timber roofs have long been appreciated by architects engaged in restoration. Certainly Viollet-le-Duc had a fundamental understanding of the problems, and the contemporary work of G. G. Scott shows the same grasp of essentials. Both would have used enthusiastically modern materials and methods, as would, indeed, the original builders.

To replace the wall plates at Westminster, for example, there are now new reinforced-concrete ring beams connected at intervals by steel joist ties. These exactly realize the intention of the mediaeval designers (an intention that they failed to realize themselves in using timber wall plates inadequately connected to timber ties) of relieving the masonry walls of any outward thrust from the roof.

With absolutely firm foundations provided in this way to the feet of the rafters, the basically good triangulation

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of the trusses will ensure a satisfactory structure, which however, still requires stiffening longitudinally. Here the former Surveyor to the Fabric, Mr. S. E. Dykes Bower, has devised for the nave and transepts an ingenious and economical system. Instead of the sheeting battens being fixed horizontally, a form of construction which maintains the trusses at fixed distances apart but allows them to sway, the battens have been fixed diagonally to the backs of the rafters, affording a sort of continuous triangulation in the third dimension, and making the whole roof structure extremely stiff. These diagonal battens, while capable of carrying quite large forces, are merely stiffeners for the nave and transept roofs, and are effectively unstressed.

Essentially the same system of diagonal battens has been used in the presbytery roof to carry the active thrust from the croupe. As was mentioned, this roof has been carefully restored rather than renovated, and has moreover been restored to its original state of a poor engineering structure. The westward thrust is, however, now carried permanently in the surface of the rafters, that is, in the skin of the roof, rather than by internal members bracing the ferme-maitresse; some additional strutting has been provided to help with the transfer of the thrust.

NOTES

- ROYAL COMMISSION ON HISTORICAL MONUMENTS (Eng-1. land). London, vol. 1: Westminster Abbey. London, H.M.S.O., 1924. Shortly after first publication an addendum slip was inserted (p. 58, entry no. 22a) giving a very brief description of the roofs.
- 2. Examples at hand for the author are: Nicolas Fourneau, L'art du trait de charpenterie, Rouen, 1767; J. Ch. Krafft, Plans, coupes. et élévations de diverses productions de l'art de la charpente, Paris, 1805, which gives examples of foreign as well as French work; and, of course, Viollet-le-Duc's short article Charpente in the third volume of the Dictionnaire. In about 1960 the Centre de Recherches sur les Monuments Historiques published in Paris a seven-volume work on Charpentes. In England Thomas Tredgold was interested more in the engineering than the comparative historical aspects of timber construction; see the third edition of Elementary principles of carpentry,

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1840, edited by Peter Barlow. Cecil Hewett, English Cathedral Carpentry, London, 1974, breaks a great deal of new ground, and he has splendid drawings showing overall structure and constructional details.

- 3. R. W. McDowall, J. T. Smith and C. F. Stell, "Westminster Abbey; The timber roofs of the Collegiate Church of St. Peter at Westminster," *Archaeologia*, 100, 1966, 155-74.
- 4. C. A. Hewett (op. cit.) has four pages on apse roofs, and he states that only four such roofs exist today. That at Ely covers the chapel of St. Catherine, is very small, and is nineteenth-century (probably designed by Robert Willis). The roof at Pershore Abbey was restored by Scott in 1862-65. Finally, the fourteenth-century Lady Chapel at Lichfield has an apse, and the roof was rebuilt in 1661-69.
- 5. See for example the second edition of H. R. Hahnloser, Graz. 1972.
- 6. Paul Frankl, The Gothic: Literary sources and interpretations through eight centuries, Princeton University Press, 1960.
- 7. Viollet-le-Duc, loc. cit.

Photograph credit: Hollar drawing (courtesy of the Victoria and Albert Museum)

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Ur Jennings, Alan (2004), *Structures: from theory to practice*, Spon Press, Oxfordshire, s. 609 - 610.

in = inch ft = foot lb = pound1 ft = 12 in

Conversion factors

1 in = 25,4 mm 1 ft = 0,3048 m

 $1 \text{ in}^2 = 645 \text{ mm}^2$

1 lb mass = 0,4536 kg 1 lb force = 4,448 N